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Multiple Contracts with Simple Interest: the Case of "Método Americano"

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INTRODUCTION

In a previous paper de Faro (2021), focusing attention on the case of the so called "Método Americano" (American Method), where the borrower is required to periodically pay interest only, with the principal of the loan being repaid by a lump-sum at the end of the *n*-th period of the loan term, it was shown that if a single contract is substituted by multiple contracts, one for each payment of the single contract, the financing institution may derive substantial fiscal gains.

However, the analysis was made under the assumption that the contract was written in terms of compound interest.

Given that the Brazilian Jurisprudence, cf Jusbrasil (2023), has repeatedly determined that compound interest implies the occurrence of anatocism, payment of interest on interest, the analysis should also consider the use of simple interest.

At this point, it should be noted that the occurrence or not of anatocism is still a debated subject, as can be seen in the arguments recently presented in Puccini (2023) and in De-Losso et al. (2023).

Circumventing the controversy, it will be assumed that both creditors and debtors have freely accepted the use of simple interest.

USING SIMPLE INTEREST

Considering a loan *F*, suppose that it must be repaid by *n* periodic payments.

If simple interest is used, at the periodic interest rate *i*, the first step is to consider the necessity of specifying what is called a focal date; cf. Ayres (1963). As, in opposition to the case of compound interest, different focal dates imply different results.

We are going to consider two distinct focal dates. The first one, time zero, which should be considered as the most natural, and is the one established in a Brazilian Law of 1964, cf. De-Losso et al. (2020), implies that the *k*-th periodic payment, denoted as p_k, must be such that:

$$F = \sum_{k=1}^{n} \frac{p_k}{1 + i \times k} \tag{1}$$

The second one, time *n* which has been repeatedly specified in the case of constant payments, as in Nogueira (2013), and in the case of constant amortization, as in Rovina (2009), implies that the corresponding periodic payments, now denoted as $\hat{p}_k$, must be such that:

$$F \times (1 + i \times n) = \sum_{k=1}^{n} \hat{p}_k \times \{1 + i \times (n - k)\} \tag{2}$$

CAPITALIZED & NON-CAPITALIZED COMPONENTS

According to Forger (2009), whenever simple interest is to be considered, the first step is to divide the loan *F* into two components: F^C and F^N, where F^C denotes the so called capitalized component, and F^N denotes the non-capitalized component.

To this end, Forger (2009) introduced a weigh factor *f*, which decomposes the loan amount *F* in such a way that:

$$F = F^C + F^N = [f \times F] + [(1 - f) \times F] \tag{3}$$

with

$$0 \le f \le 1 \tag{4}$$

and f being dependent on the particular system of amortization under consideration, as well as, on the focal date.

Additionally, not depending on a particular system of amortization being considered, it is supposed that the parcel of interest that is included in the *k*-th periodic payment, denoted as J_k, is such that:

$$J_k = i \times F_{k-1}^C \quad , \text{ for } k = 1, 2, \ldots, n \tag{5}$$

In what follows, in order to give a numerical example, which will be used for both focal dates, we are going to assume that $F = \$120,000.00$, $n = 12$ periods, and that the periodic rate of simple interest is $i = 1\%$ per period.

FOCAL DATE AT TIME ZERO

Let us, initially, consider the case of a single contract using the American Method. If the date where the loan granted is taken as the focal date, time zero, the first step is to determine the value of the weigh factor *f*.

For the case under scrutiny, where we have payment of interest only, it was determined in Lachtermacher and de Faro (2022), that the value of f is given by the following expression:

$$f = \frac{n}{1+i\times n}\times\left\{\sum_{k=1}^{n}\left[\frac{1}{1+i\times k}\right]\right\}^{-1} \quad (6)$$

Although being a rather complex expression, its solution is easily accomplished using a spreadsheet, such as Excel.

In the case of our numerical example, where $n = 12$ periods, and the rate of simple interest is $i = 1\%$ per period, it follows that $f = 0.949892984$.

In Table 1 we have the sequence of the parcels of interest, the sequence of the periodic payments, and the evolution of the outstanding debt S_k.

Table 1. Evolution of the Debt on the Case of a Single Contract - Focal Date Time Zero

k	J_k	P_k	S_k
0			120,000.00
1	1,139.87	1,139.87	120,000.00
2	1,139.87	1,139.87	120,000.00
3	1,139.87	1,139.87	120,000.00
4	1,139.87	1,139.87	120,000.00
5	1,139.87	1,139.87	120,000.00
6	1,139.87	1,139.87	120,000.00
7	1,139.87	1,139.87	120,000.00
8	1,139.87	1,139.87	120,000.00
9	1,139.87	1,139.87	120,000.00
10	1,139.87	1,139.87	120,000.00
11	1,139.87	1,139.87	120,000.00
12	1,139.87	121,139.87	0.00
Σ	13,678.46	133,678.46	

Suppose now that a single contract is substituted by n individual contracts - one for each payment of the single contract.

In this event, adapting to the case of simple interest the methodology proposed in De-Losso et al. (2013), the principal of the k-th subcontract, denoted as F_k, is taken to be equal to the present value of the k-th payment of the single contract. That is:

$$F_k = \frac{p_k}{1+i\times k} \quad , \text{ for } k = 1,2,\ldots,n \quad (7)$$

with the corresponding parcel of interest, now denoted as J_k', being such that:

$$J_k' = \frac{p_k \times i\times k}{1+i\times k} \quad , \text{ for } k = 1,2,\ldots,n \quad (8)$$

In Table 2, we show what can be regarded as the consolidation of all the twelve subcontracts.

Additionally, it is also presented in Table 2 the sequence of differences $d_k = J_k - J_k'$; for $k = 1,2,\ldots,n$.

Table 2. Consolidation of the Subcontracts: Focal Date Time Zero

k	F_k	p_k	J_k'	d_k
1	1,128.59	1,139.87	11.29	1,128.59
2	1,117.52	1,139.87	22.35	1,117.52
3	1,106.67	1,139.87	33.20	1,106.67
4	1,096.03	1,139.87	43.84	1,096.03
5	1,085.59	1,139.87	54.28	1,085.59
6	1,075.35	1,139.87	64.52	1,075.35
7	1,065.30	1,139.87	74.57	1,065.30
8	1,055.44	1,139.87	84.43	1,055.44
9	1,045.75	1,139.87	94.12	1,045.75
10	1,036.25	1,139.87	103.62	1,036.25
11	1.026.91	1,139.87	112.96	1,026.91
12	108,160.60	121,139.87	12,979.27	-11,839.40
Σ		133,678.46	13,678.46	0.00

Strictly from an accounting point of view, there is no gain for the financial institution granting the loan if a single contract is substituted by multiple contracts, since the sum of the corresponding parcels of interest is the same.That is:

$$\sum_{k=1}^{12} J_k = \sum_{k=1}^{12} J_k' = \$13,678.46$$

Yet, depending on the opportunity cost of the financial institution, which will be denoted as ρ, and is usually of compound interest, and which is supposed to be relative to the same period of the simple interest rate i that is being charged, the financial institution may derive substantial gains in terms of tax deductions.

In other words, it is possible that:

$$V_1(\rho) = \sum_{k=1}^{n} J_k \times (1+\rho)^{-k} > V_2(\rho) = \sum_{k=1}^{n} J_k' \times (1+\rho)^{-k} \quad (9)$$

where $V_1(\rho)$ denotes the present value, at the rate ρ of the sequence of the parcels of interest in the case of a single contract, and $V_2(\rho)$ denotes the corresponding present value in the case of the adoption of multiple contracts.

Moreover, taking into account that the sequence d_k of differences has only one change of sign, thus characterizing what is termed a conventional financing project, cf. de Faro (1974), whose internal rate of return is unique, and in this particular case is null, it follows that $\Delta = V_1(\rho) - V_2(\rho) > 0$ if $\rho > 0$.

Taking into account that in Brazil the monthly interest rates charged do not exceed 2% per month, in real terms, we are going to analyze the behavior of the percentage increase of the fiscal gain $\delta = [V_1(\rho_a)/V_2(\rho_a) - 1]\times 100$ for some values of the corresponding annual opportunity cost ρ_a, with each contract with a term of n_a years. This is depicted in Tables 3, 4, 5 and 6.

Table 3. Fiscal gain δ when $i = 0.5\%$ monthly – focal date time zero

i=0.5% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	11.7556	24.6609	38.7747	54.1509	70.8376	88.8763
10	23.2958	51.8384	86.2350	126.9039	173.9886	227.2956
15	34.3761	79.8792	137.3565	206.0563	283.4890	365.9758
20	44.9165	107.1271	185.2826	273.5276	364.0916	450.8445
25	54.8279	131.9495	224.1883	319.0551	407.4677	486.7389
30	64.0291	153.1637	251.9598	344.7214	426.5294	499.1644

Table 4. Fiscal gain δ when $i = 1\%$ monthly – focal date time zero

i=1% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	10.7808	22.4400	34.9913	48.4408	62.7866	78.0183
10	19.9972	43.3657	70.0982	99.9859	132.6215	167.4347
15	27.9238	61.8549	100.9139	143.4268	187.3631	230.8681
20	34.7929	77.4258	124.7594	172.8273	218.6067	260.7191
25	40.7472	89.9115	141.2436	189.5356	232.8850	271.6484
30	45.8915	99.4816	151.6670	197.9745	238.6278	275.1836

Table 5. Fiscal gain δ when $i = 1.5\%$ monthly – focal date time zero

i=1.5% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	9.9986	20.6830	32.0427	44.0602	56.7103	69.9616
10	17.6954	37.7084	59.8287	83.7072	108.8916	134.8789
15	23.8680	51.3758	81.4100	112.5510	143.4547	173.1340
20	28.9411	62.0309	96.5581	130.0253	161.0155	189.1736
25	33.1554	70.0639	106.3043	139.2657	168.5305	194.7227
30	36.6707	75.9316	112.1703	143.7378	171.4521	196.4706

Table 6. Fiscal gain δ when $i = 2\%$ monthly – focal date time zero

i=2% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	9.3518	19.2466	29.6609	40.5650	51.9234	63.6963
10	15.9713	33.6006	52.6101	72.6363	93.2730	114.1169
15	21.0258	44.3862	69.0221	93.8085	117.8308	140.5192
20	25.0383	52.3935	79.8742	105.8185	129.4948	150.8855
25	28.2843	58.2125	86.5754	111.9202	134.3103	154.3632
30	30.9353	62.3481	90.5010	114.8061	136.1484	155.4427

As indicated, the fiscal gains are highly significant.

FOCAL DATE AT THE END OF THE CONTRACT

As in the previous section, let us start the analysis with the case of a single contract.

If the focal date is the end of the contract, time *n*, it was shown in Lachtermacher and de Faro (2022) that the weigh factor *f* now is equal to:

$$f = \frac{2n}{2n + i \times n \times (n-1)} \qquad (10)$$

Therefore, in the case of our numerical example, we have that $f = 0.947867299$.

In Table 7, we have the sequences of the parcels of interest $\hat{J}_k$ and of the periodic payments $\hat{p}_k$, as well as the evolution of the outstanding debt.

Table 7. Evolution of the Debt in the Case of a Single Contract: focal date time *n*

k	$\hat{J}_k$	$\hat{p}_k$	S_k
0			120,000.00
1	1,137.44	1,137.44	120,000.00
2	1,137.44	1,137.44	120,000.00
3	1,137.44	1,137.44	120,000.00
4	1,137.44	1,137.44	120,000.00
5	1,137.44	1,137.44	120,000.00
6	1,137.44	1,137.44	120,000.00
7	1,137.44	1,137.44	120,000.00
8	1,137.44	1,137.44	120,000.00
9	1,137.44	1,137.44	120,000.00
10	1,137.44	1,137.44	120,000.00
11	1,137.44	1,137.44	120,000.00
12	1,137.44	121,137.44	0.00
Σ	13,649.29	133,649.29	

As in the case of focal date at time zero, we have the same behavior regarding the evolution of the outstanding debt.

On the other hand, if a single contract is substituted by *n* subcontracts, we must have an adaptation of the procedure suggested in De-Losso et al. (2013).

In this case, the principal of the *k*-th subcontract, now denoted as $\hat{F}_k$, must be such that:

$$\hat{F}_k = \hat{p}_k \times \left[1 + i \times (n-k) / (1 + i \times n)\right] \quad , k = 1, 2, \ldots, n \tag{11}$$

as we must have $\sum_{k=1}^{n} \hat{F}_k = F$.

Table 7 depicts the sequences of principals $\hat{F}_k$, of payments $\hat{p}_k$, of the parcels of interest $\hat{J}'_k$, as well of the sequence of differences $\hat{d}_k = \hat{J}_k - \hat{J}'_k$

Table 8. Consolidation of the subcontracts: focal date time *n*

k	$\hat{F}_k$	$\hat{p}_k$	$\hat{J}'_k$	$\hat{d}_k$
1	1,127.29	1,137.44	10.16	1,127.29
2	1,117.13	1,137.44	20.31	1,117.13
3	1,106.97	1,137.44	30.47	1,106.97
4	1,096.82	1,137.44	40.62	1,096.82
5	1,086.66	1,137.44	50.78	1,086.66
6	1,076.51	1,137.44	60.93	1,076.51
7	1,066.35	1,137.44	71.09	1,066.35
8	1,056.19	1,137.44	81.25	1,056.19
9	1,046.04	1,137.44	91.40	1,046.04
10	1,035.88	1,137.44	101.56	1,035.88
11	1,025.73	1,137.44	111.71	1,025.73
12	108,158.43	121,137.44	12,979.01	-11,841.57
Σ	120,000.00	133,649.29	13,649.29	0.00

Once more, from an accounting point of view, there is no fiscal gain if the financial institution substitutes a single contract by multiple contracts, since:

$$\sum_{k=1}^{12} \hat{J}_k = \sum_{k=1}^{12} \hat{J}'_k = \$13,649.29$$

However, similarly to the case of focal date time zero, there is the possibility of substantial fiscal gains if the financial institution substitutes a single contract by *n* subcontracts.

That is, in general, we have that:

$$V_3(\rho)=\sum_{k=1}^{n}\hat{J}_k\times(1+\rho)^{-k}>V_4(\rho)=\sum_{k=1}^{n}\hat{J}_k^{'}\times(1+\rho)^{k} \quad (9)$$

whenever $\rho>0$.

Numerical evidence of the magnitude of the percentage increase of the corresponding fiscal gain $\delta'=\left[V_4(\rho)/V_3(\rho)-1\right]\times 100$, is provided in Tables 9,10,11 and 12.

Table 9. Fiscal gain δ' when $i=0.5\%$ monthly – focal date time *n*

i=0.5% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	11.9059	25.0103	39.3818	55.0859	72.1833	90.7293
10	24.3333	54.6514	91.9139	136.9818	190.5119	252.8508
15	37.4864	89.4919	159.0758	247.9876	355.7336	479.2240
20	51.5960	130.0529	240.5544	381.7579	544.5375	715.7819
25	66.8097	176.3725	333.1083	523.8626	725.0301	918.2650
30	83.2246	227.9667	431.3911	660.4091	882.2716	1085.0502

Table 10. Fiscal gain δ' when $i=1\%$ monthly – focal date time *n*

i=1% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	11.2190	23.4432	36.7080	51.0424	66.4681	82.9994
10	22.4713	49.7861	82.4473	120.7822	164.8774	214.5372
15	34.3431	80.2259	139.0514	210.8632	294.0242	385.4630
20	47.1203	115.3995	206.5706	317.2214	439.5802	564.9741
25	60.9496	155.4429	283.0172	431.3471	584.1286	730.5296
30	75.9187	200.0703	365.0383	545.0442	718.8958	879.7336

Table 11. Fiscal gain δ' when $i=1.5\%$ monthly – focal date time *n*

i=1.5% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	10.7538	22.3934	34.9369	48.3951	62.7714	78.0618
10	21.4321	47.1410	77.4457	112.4823	152.1628	196.1612
15	32.7842	75.8112	129.9255	194.7176	268.4076	348.2063
20	45.0719	109.0317	192.6058	292.1004	400.5936	510.9577
25	58.4198	146.9425	263.8875	397.7926	534.8463	666.3101
30	72.9024	189.3044	340.9497	504.8943	663.3492	810.7029

Table 12. Fiscal gain δ' when $i=2\%$ monthly – focal date time *n*

i=2% monthly	ρa(%)					
n_a	5%	10%	15%	20%	25%	30%
5	10.4179	21.6411	33.6774	46.5273	60.1845	74.6350
10	20.7691	45.4786	74.3529	107.4364	144.5666	185.3733
15	31.8529	73.2284	124.7041	185.6858	254.3836	328.2031
20	43.8974	105.4709	184.9957	278.7327	380.2523	483.1875
25	57.0097	142.3344	253.7904	380.4550	509.7468	633.8835
30	71.2553	183.5965	328.4983	484.4858	635.3673	776.0755

Once more, the fiscal gain is substantial.

CONCLUSION

Analogously to the case of compound interest, the adoption of simple interest in the American Method of Payment ("método americano") also implies the possibility of substantial fiscal gain for the financial institution, if a single contract is substituted by n subcontracts, this result being observed regardless of the choice of the focal date.

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