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A Comparison of the German and Tedesco Systems of Amortization

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ABSTRACT

Although not very popular neither in Brazil nor in Italy, the German and Tedesco systems of debt amortization have been discussed both in the Brazilian specialized literature, as well in the corresponding Italian literature.

Both methods are characterized by the payment of the interest in advance, at the beginning of the period, and constant installments. However, they have different ways of determining the interest, amortization and installment.

In this article we highlight the differences and compare both methods to the so-called French methods to determine which of them is a better option for the financial institution providing the loan.

KEYWORDS: *German Amortization Method; Tedesco Amortization Method; Compound Interest*

INTRODUCTION

Although not very popular neither in Brazil nor in Italy, these two systems of debt amortization have been discussed both in the Brazilian literature specialized in Mathematics of Finance, as well in the corresponding Italian literature. Both characterized by charging interest in advance. That is, at the beginning of each period. Instead of at the end of the period as it is the usual practice.

In Brazil, for instance, the so-called German Method of Amortization have been studied in Moraes (1967), in Juer (2003) and de Faro & Lachtermacher(2012). While in Italy, where is known as "L'Ammortamento Tedesco", is discussed in Palestini (2017), in class-notes presented in the Sapienza Università di Roma.

As there are significant differences in their respective methodologies, it appears to be relevant to address them.

Furthermore, as both methodologies make use of constant installments, besides an initial payment, a comparison with the classical schema of constant payments is also presented.

THE BRAZILIAN APPROACH

Consider the case where a loan in the amount of F units of capital must be repaid at the periodic rate i of compound interest, in a contract with a term of n periods, written in accordance with the German Method of Amortization. As it is known in Brazil.

As the first payment is in advance, the borrower has pay, at the very first day of the contract, an initial payment, denoted as P_0, whose value is $P_0 = i \times F$.

Additionally, the borrower must pay n periodic installments, P_k, for $k = 1, 2, \ldots, n$, with a constant value equal to$P$. Each of them having to be paid at the end of the k^{th} period.

As the borrower receives only the amount$F\times(1 - i)$, the value of P, can be derived assuming the periodic interest rate i^*, that implies the solution of the classical expression of the case of constant payments, cf, de Faro and Lachtermacher (2012):

$$F(1-i) = P \times \left\{ \frac{1-(1+i^*)^{-n}}{i^*} \right\} \tag{1}$$

or

$$P = \frac{F \times (1-i) \times i^*}{1-(1+i^*)^{-n}} \tag{1'}$$

In principle, the solution of equation (1) would require a process of trial and error. As both the values of P and of i^* are unknown. However, this not necessary, since an exact solution is given by:

$$i^* = i/(1-i) \tag{2}$$

From which follows that:

$$P = \frac{F \times i}{1-(1-i)^n} \tag{3}$$

Alternatively, denoting by S_k the outstanding debt at time k, just after the payment P_k, by A_k the parcel of amortization at time k, and by $J_k = i \times S_k$ the parcel of interest, with A_k and J_k being the components of P_k we have:

$$P_k = A_k + J_k = A_k + i \times S_k \text{ , for } k = 0,1,\ldots,n \quad (4)$$

It should be noted that, in this case, where the first payment is of interest only, which means that $J_0 = i \times S_0$, it follows that $A_k = 0$. That is $J_0 = i \times F = P_0$. Which means that $J_0 = F$.

On the other hand, as it is assumed that $P_k = P$, for $k=1,2,\ldots,n$ it follows that:

$$A_1 + i \times F = A_2 + i \times (F - A_1 - A_2)$$

so that

$$A_2 = A_1 / (1 - i) \quad (5)$$

That is, as can be proven by induction, the parcels of amortization follow a geometric sequence with ratio $1/(1-i)$.

Therefore, considering that, by definition,

$$S_k = F - \sum_{\ell=0}^{k} A_\ell = F - \sum_{\ell=1}^{k} A_\ell \quad \text{since } A_0 = 0 \quad (6)$$

we have, trivially

$$S_n = 0 \Rightarrow F = \sum_{\ell=1}^{n} A_\ell$$

Thus, considering the expression of the sum of the first elements of a geometric progression with initial term equal to $A_1$and ratio $1 / (1 + i)$we have:

$$F = \frac{A_1 - (1-i)^{-1} \times (1-i)^{1-n} \times A_1}{1 - (1+i)^{-1}}$$

Therefore

$$A_1 = \frac{i \times F \times (1-i)^n}{(1-i) \times \left[1 - (1-i)^n\right]} \quad (7)$$

from which follows that

$$A_n = A_1 \times (1-i)^{1-n} \quad (8)$$

With $A_n = P$, since $J_n = 0$.

As a simple numerical illustration, consider the case where F = 100,000.00 units of capital, the financing rate is equal to 1% per period, and the term of the contract is n=12.

The first payment is P_0 = 1,000.00 and the constant payment is P = 8,801.64.

Table 1 shows the evolution of the debt. Being worth noticing that besides the column of the outstanding debt S_k. Where S_k, is the outstanding debt at the end of each period.

Table 1. Evolution of the Debt – Brazilian Way

Epoch (k)	J_k	A_k	P_k	S_k
0	1,000.00	0.00	1,000.00	100,000.00
1	921.20	7,880.45	8,801.64	92,119.55
2	841.60	7,960.05	8,801.64	84,159.50
3	761.19	8,040.45	8,801.64	76,119.05
4	697.97	8,121.67	8,801.64	67,997.38
5	597.94	8,203.71	8,801.64	59,793.67
6	515.07	8,286.57	8,801.64	51,507.09
7	431.37	8,370.28	8,801.64	43,136.82
8	346.82	8,454.82	8,801.64	34,681.99
9	261.42	8,540.23	8,801.64	26,141.77
10	175.15	8,626.49	8,801.64	17,515.27
11	88.02	8,713.63	8,801.64	8,801.64
12	0.00	8,801.64	8,801.64	0.00
∑	6,619.74	100,000.00	106,619.74	

To contrast to what will be presented in the next section, it appears to be useful to show the steps that would be required when using a financial calculator, such as a HP 12C.

Setting the display at the end mode, consider the case of our numerical example.

Starting with 100,000 × (1 – 0.01) = 99,000, n=12 and $i^* = 0.01/(1-0.01) = 0.010101$, multiply it by 100, so i=1,010101%p.p. Enter the following sequence of commands in your HP12C (where each keyboard is between brackets).

[g][end][f][REG]99000[PV]12[n]1.010101[i][PMT]

appear at the display -8,801.644767

Which is the constant installment of the German system.

THE ITALIAN APPROACH

As in the previous section, consider a loan of F units of capital, at the periodic rate i of compound interest, with a term of n periods.

According to Palestini (2017), the socalled "l'ammortamento Tedesco", implies an initial payment, at the very date of the issuance of the contract, denoted as P'_0, given by:

$$P'_0 = i \times F / (1+i) \quad (9)$$

With the n remaining payments P'_k, assumed to be constant and equal to P', being determined in accordance with the concept of an annuity-due, cf. Kellison (1970).

That is, considering the full amount F of the loan, we will have:

$$P' = \frac{F \times i}{(1+i) \times \left[1 - (1+i)^{-n}\right]} \quad (10)$$

However, it should be stressed that, contrary to the classical concept of an annuity-due, the first of these constant payments is supposed to occur at the beginning of the second period (or end of the first period). And not at the beginning of the first period which is the general case.

According to Palestini (2017), we also have the recursion:

$$S'_k = (S'_{k-1} - P'_k) \times (1+i), \quad k = 1,2,\ldots,n \quad (11)$$

Where S'_k, is the outstanding debt at time k, with $S'_0=F$, $S'_n=0$ and

$$P'_k = \begin{cases} i \times F / (1+i), & \text{if } k = 0 \\ P' & \text{if } k = 1,2,\ldots,n \end{cases} \quad (12)$$

Furthermore, it is also established that:

$$A'_k = \frac{P'}{(1+i)^{n-k}}, \quad \text{for } k = 1,2,\ldots,n \quad (13)$$

with $A'_0 = 0$, and

$$J'_k = P'_k - A'_k, \quad \text{for } k = 0,1,\ldots,n \quad (14)$$

If should be observed that, the sequence of parcels of amortization follows a geometric sequence with ratio $1+i$.

Considering the same numerical example of the previous section, Table 2 presents the corresponding evolution of the debt.

Table 2. Evolution of the Debt – Italian Way

Epoch (k)	J^I_k	A^I_k	P^I_k	S^I_k
0	990,10	0.00	990,10	100.000,00
1	912,03	7.884,88	8.796,91	92.115,12
2	833,18	7.963,73	8.796,91	84.151,39
3	753,54	8.043,36	8.796,91	76.108,03
4	673,11	8.123,80	8.796,91	67.984,23
5	591,87	8.205,04	8.796,91	59.779,19
6	509,82	8.287,09	8.796,91	51.492,11
7	426,95	8.369,96	8.796,91	43.122,15
8	343,25	8.453,66	8.796,91	34.668,49
9	258,72	8.538,19	8.796,91	26.130,30
10	173,33	8.623,58	8.796,91	17.506,72
11	87,10	8.709,81	8.796,91	8.796,91
12	0,00	8.796,91	8.796,91	0,00
∑	6.553,02	100,000.00	106.553,02	

As can be observed by comparing Tables 1 and 2, both the Tedesco method and the German method charge the interest in advance. However, with different ways of calculating the installments and interest's parcels. Resulting that the Tedesco method presents a smaller total of interest and smaller constant installments than in the German method.

In this case, using the same financial calculator HP12C, we should press the following commands:

[g][BEG][f][REG]100000[PV]12[n]1[i][PMT]

appear at the display -8,796.909770

Which is the constant installment, *P'* of the Tedesco System.

COMPARISON WITH THE FRENCH SYSTEM

Considering that, according to Annibali et al.(2016), the classical amortization system of constant payments is also named as the French System, it appears appropriated to make a comparison of these three somewhat similar amortization systems.

Considering our simple numerical example, Table 3 presents the evolution of the debt if the French system is implemented.

Table 3. French Amortization Method – evolution of the debt

Epoch (k)	$\hat{J}_k$	$\hat{A}_k$	$\hat{P}$	$\hat{S}_k$
0				100,000.00
1	1,000.00	7,884.88	8,884.88	92,115.12
2	921.15	7,963.73	8,884.88	84,151.39
3	841.51	8,043.36	8,884.88	76,108.03
4	761.08	8,123.80	8,884.88	67,984.23
5	679.84	8,205.04	8,884.88	59,779.19
6	597.79	8,287.09	8,884.88	51,492.11
7	514.92	8,369.96	8,884.88	43,122.15
8	431.22	8,453.66	8,884.88	34,668.49
9	346.68	8,538.19	8,884.88	26,130.30
10	261.30	8,623.58	8,884.88	17,506.72
11	175.03	8,709.81	8,884.88	8,796.91
12	87.97	8,796.91	8,884.88	0.00
∑	6,618.19	100,000.00	106,618.55	

From Table 3 we see that the corresponding value of the constant payment is $\hat{P} = \$8,884.88$ units of capital. A value that is only 0.946% greater than the corresponding one in the case of the German method and 1.00% greater than the Tedesco method.

Furthermore, from the strict accounting point of view, there is no significant difference in terms of the total of interest payments. As the total of interest in the case of the German system is only 0.02% greater than the corresponding one in the case the French system and 0.99% greater than the Tedesco method.

A result that is always observed. As confirmed in Table 4, for the cases where *F*=$100,000.00 units of capital, the financing interest *i* takes the values of 0.5%, 1% and 2% per period, and the number *n* of periods varies from 12 to 360.

Table 4. Percentage of the total of interest paid over the loan

n	German Amortization System			Tedesco Amortization System			French Amortization System		
	0.50%	1.00%	2.00%	0.50%	1.00%	2.00%	0.50%	1.00%	2.00%
12	3.280	6.620	13.481	3.263	6.553	13.207	3.280	6.619	13.472
60	16.001	33.496	72.831	15.917	33.135	71.184	15.997	33.467	72.608
120	33.239	72.277	165.313	33.059	71.451	161.350	33.225	72.165	164.577
180	51.927	116.262	271.741	51.636	114.881	265.186	51.894	116.030	270.489
240	71.999	164.629	385.792	71.586	162.634	376.645	71.943	164.261	384.178
300	93.374	216.471	503.403	92.826	213.829	491.747	93.290	215.967	501.582
360	115.954	270.926	622.500	115.262	267.624	608.409	115.838	270.301	620.578

This confirm our previous finding that the total amount of interest of the German method is greater than the French method. Which is greater than the Tedesco method.

However, a more relevant comparison must take into consideration the financial institution cost of capital. Which periodic value will be denoted as ρ.

That is, we must compare the present values of the corresponding sequences of the parcels of interest payments. Respectively designated as $V_1(\rho)$, for the German method, $V_2(\rho)$, for the Tedesco method and $V_3(\rho)$, for the French method, given by:

$$V_1(\rho) = \sum_{k=0}^{n} J_k \times (1+\rho)^{-k}$$

$$V_2(\rho) = \sum_{k=0}^{n} J'_k \times (1+\rho)^{-k}$$

$$V_3(\rho) = \sum_{k=1}^{n} \hat{J}_k \times (1+\rho)^{-k}$$

where ρ is supposed be relative to the same period as the financing interest rate i.

For instance, if ρ_a is the financial institution cost of capital, in annual terms, is equal to 20%, which means that ρ=1,531% per month, n=120 periods, and interest rate i=1% per month, and F=100,000.00, we have $V_1(\rho)$=41,008.80, $V_2(\rho)$=40,557.84 and $V_3(\rho)$=40,345.75 units of capital.

Which implies that the financial institution, in terms of the payment of interest, will earn more, if the loan is implemented with the German method, instead of the Tedesco method or the French method.

This conclusion is valid for every positive value of the rate ρ. Tables 5, 6 and 7 show the results for i=1% per month, F=100,000.00, n=120, 240 and 360 months and ρ_a varying from 5% to 30% annually.

Table 5

n=120, i=1%p.m, F=100,000				
ρ_a	ρ	$V_1(\rho)$	$V_2(\rho)$	$V_3(\rho)$
5%	0.407%	61,018.52	60,328.80	60,684.85
10%	0.797%	52,576.29	51,988.05	52,092.54
15%	1.171%	46,093.48	45,582.47	45,505.20
20%	1.531%	41,008.80	40,557.84	40,345.75
25%	1.877%	36,944.76	36,541.41	36,226.88
30%	2.210%	33,641.33	33,276.40	32,882.32

Table 6

n=240, i=1%p.m, F=100,000				
ρ_a	ρ	$V_1(\rho)$	$V_2(\rho)$	$V_3(\rho)$
5%	0.407%	116,116.50	114,755.29	115,432.56
10%	0.797%	87,488.91	86,491.03	86,664.86
15%	1.171%	69,423.21	68,648.71	68,532.34
20%	1.531%	57,341.58	56,712.97	56,416.40
25%	1.877%	48,852.45	48,324.23	47,908.27
30%	2.210%	42,633.74	42,177.75	41,678.25

Table 7

n=360, i=1%p.m, F=100,000				
ρ_a	ρ	$V_1(\rho)$	$V_2(\rho)$	$V_3(\rho)$
5%	0.407%	159,634.79	157,792.33	158,723.60
10%	0.797%	107,605.58	106,413.00	106,626.87
15%	1.171%	79,846.23	78,985.77	78,851.88
20%	1.531%	63,329.50	62,659.79	62,332.11
25%	1.877%	52,615.06	52,065.73	51,617.57
30%	2.210%	45,182.12	44,714.59	44,185.05

It should be noted that if comparing the Tedesco method and the French Method, a definitive conclusion cannot be reached since the greater present value vary depending on the cost of capital of the financial institution.

CONCLUSION

Comparing the Brazilian approach for the implementation of the so-called German Method of Amortization, with the Italian approach, it was shown that they result in stark differences.

In all cases that were considered it was observed that the financial institution providing the loan should always prefer to choose the implementation of the "Brazilian approach". Since it will derive greater revenue in terms of payments of interest.

Additionally, it was concluded that with the possibility of implementing the French system, the financial institution is better off if sticks to the choice of the "Brazilian approach".

In comparing the Tedesco method with the French method, the financial institution providing the loan, has no unique preference, since it will depend on its cost of capital, interest rate and term of the loan.

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