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Portfolio Optimization: Exploring Markowitz Models and Modern Approaches for Effective Frontier Analysis

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ABSTRACT

The article discusses approaches to optimizing investment portfolios with an emphasis on the Markowitz model, as well as other mathematical methods. As the purpose of the work, it was chosen to study algorithms for building portfolios, analyze the effective front, identify key characteristics of models, and identify their limitations.

The materials of the article are based on methods including the Markowitz model, stochastic algorithms, and evolutionary approaches. Covariance matrices, forecasting returns, accounting for asymmetric risks, and the impact of market volatility are studied. A comparative analysis of classical methods and modern solutions, such as robust optimization, machine learning, is carried out.

The conclusions of the article confirm the applicability of the Markowitz model in stable market conditions with rational strategies of participants. However, market instability creates limitations for the use of this model. Modern approaches demonstrate adaptability, resistance to uncertainty, and require significant computing resources.

The information reflected in the framework of the work, based on the study of literary sources, will be useful for studying by specialists working in the field of economics, financial analysis, asset management. The practical value of the work lies in the analysis of existing methods aimed at reducing risks, which will improve the profitability of portfolios.

KEYWORDS: *portfolio optimization, Markowitz model, effective front, asset management, stochastic methods, financial stability.*

INTRODUCTION

Portfolio optimization is a key aspect of asset management, ensuring resource allocation in line with specified financial parameters. The theory of portfolio investment, introduced by Harry Markowitz in the mid-20th century, has undergone significant transformations. The classical model has been enriched with modern analytical tools adapted to the dynamics of financial markets.

Contemporary approaches employ methods that account for a wide range of factors influencing asset behavior, enabling more accurate predictions of their returns. Despite the development of alternative concepts, Markowitz's model, based on the assumption of normally distributed returns, remains fundamental in portfolio management. Recently, stochastic algorithms, robust approaches, and machine learning methods have been actively utilized. These technologies minimize dependency on initial data, eliminate parameter rigidity, and consider atypical market events and investor behavioral characteristics.

Modern financial markets are characterized by high complexity and significant uncertainty. Innovative methods of portfolio analysis and management require consideration of numerous factors influencing asset movements. The application of new approaches facilitates the development of tools that ensure precise forecasting and effective risk management. The need to refine existing models is driven by the instability of the economic environment, which necessitates the adaptation of current methods.

The purpose of this study is to analyze the Markowitz model and examine alternative methodologies and their practical application in the context of the modern economy.

MATERIALS AND METHODS

Contemporary scientific studies on portfolio optimization based on the Markowitz model encompass a wide range of approaches, adapting the theory to various market conditions. Researchers analyze historical data, propose new analytical methods, and develop techniques to enhance the model's accuracy.

Classical methods for constructing the efficient frontier rely on retrospective data. Deng X. (2022) investigated the application of the model to the Chinese market, focusing on asset-specific characteristics. Zhang M. (2022) analyzed long-term data to evaluate the model's behavior across different economic contexts. Chen, Y., Lu, H., & Yu, T. (2023) adapted the model to the American market, accounting for asset-specific features.

Several studies compare the Markowitz model with alternative approaches. Yao L. (2023) and Wang, Y. (2023) examined traditional methods and index-based approaches, emphasizing their applicability depending on market conditions. Kim Y. S. (2020) proposed a model considering risk asymmetry and specific return distributions, tailored to real financial scenarios.

Recent research actively employs machine learning technologies and multi-objective optimization methods to improve the efficient frontier. Alexander N., Scherer W., and Burkett M. (2021) applied data dimensionality reduction techniques to forecast the efficiency frontier, simplifying the processing of large data volumes. Qi Y., Li X., and Zhang S. (2021) introduced an approach focused on asset management with constraints specific to practice. Massahi M., Mahootchi M., and Arshadi Khamseh A. (2020) developed a clustering analysis method that enhances portfolio resilience under market volatility.

The article by Kalygin V. A. and Slashchinin S. V. (2020) addresses the construction of the efficient frontier in conditions of market uncertainty, emphasizing the functioning of unstable systems. Monticeli A. R. et al. (2023) presented an approach that includes asset reallocation and experimental analysis methods to mitigate risks.

An analysis of existing research demonstrates the diversity of approaches to adapting the Markowitz model. Table 1 below outlines these adaptation approaches along with the advantages of the modified model identified by the authors.

Table 1. Adaptation Approaches and Advantages of the Modified Model (compiled by the author)

Adaptation Approach	Advantage of the Modified Model	Authors
Analysis of Historical Data and Market Specifics	Incorporation of asset specifics for the Chinese market.	Deng X. (2022)
	Evaluation of the model's behavior across various economic contexts.	Zhang M. (2022)
	Adaptation of the model to the specific features of the U.S. market.	Chen, Y., Lu, H., & Yu, T. (2023)
Comparison with Alternative Approaches	Application of models tailored to market conditions.	Yao L. (2023), Wang, Y. (2023)
	Consideration of risk asymmetry and specific return distributions.	Kim Y. S. (2020)
Use of Machine Learning Technologies	Data dimensionality reduction for simplified processing.	Alexander N., Scherer W., Burkett M. (2021)
	Asset management considering practical constraints.	Qi Y., Li X., Zhang S. (2021)
	Clustering analysis to enhance portfolio resilience.	Massahi M., Mahootchi M., Arshadi Khamseh A. (2020)
Adaptation to Market Uncertainty	Formation of the efficient frontier under instability.	Kalygin V. A., Slashchinin S. V. (2020)
	Asset reallocation and risk mitigation through experimental analysis.	Monticeli A. R. et al. (2023)

Questions regarding the applicability of the model in unstable markets remain, including the impact of long-term factors and behavioral investor models on portfolio resilience. Both theoretical and practical methods have been applied to optimize portfolios.

RESULTS AND DISCUSSION

The Markowitz model is based on mathematical methods for portfolio formation, where return and risk are quantitatively evaluated. The objective of investment portfolio optimization is to find the optimal balance between return and risk. Investors aim either to increase return at a given level of risk or to minimize risk while maintaining return. Variability in returns is interpreted as risk, and the relationships between assets are determined by the covariance matrix. Optimization is represented as a quadratic programming problem with constraints, including asset weight coefficients, covariance parameters, average return indicators, and target parameters.

Diversification involves the allocation of assets with minimal correlation, reducing overall risk. Even with significant changes in individual elements, the portfolio becomes more resilient due to proper distribution of shares. The efficient frontier describes a set of portfolios with an optimal risk-return balance. Constructing this frontier involves solving problems with varying initial parameters (Yao L. (2023) and Wang, Y. (2023)).

Errors in input data, such as incorrect estimates of correlations and risks, alter the portfolio structure, reducing its stability in real market conditions. The use of normally distributed returns simplifies calculations and excludes complex dependencies, as well as distribution characteristics that reflect rare events or atypical changes. However, financial markets are often characterized by asymmetry and extreme fluctuations, making such assumptions insufficiently accurate.

The model relies on the invariance of return and covariance parameters over a given period. In unstable markets, these assumptions do not align with real conditions. Erroneous data increase portfolio sensitivity to external factors, altering its composition.

Stochastic approaches involve analyzing various market development scenarios, making them suitable for long-term strategies. Robust methods account for errors in input data and ensure parameter stability under changing market conditions. These approaches employ interval estimates of covariances to minimize the impact of market fluctuations (Kim Y. S. (2020)). To address the noted shortcomings, robust methods and approaches based on VaR (Value at Risk) or CVaR (Conditional Value at Risk) are used. These tools take into account parameter instability and possible deviations from standard asset behavior under market uncertainty.

Table 2 outlines the advantages and limitations of stochastic approaches in creating an optimal portfolio and calculating the efficient frontier.

Table 2. Advantages and Limitations of Stochastic Approaches in Portfolio Optimization and Efficient Frontier Calculation (compiled by the author)

Parameter	**Advantages**	**Limitations**
Uncertainty Accounting	- Models the impact of uncertainty on returns and risks.	- Requires a large volume of data for reliable modeling.
Flexibility	- Captures nonlinear dependencies between assets.	- Complex to implement due to the need for specialized algorithms.
Adaptability	- Accounts for changing market conditions.	- Results depend on the accuracy of assumptions about return distributions.
Comprehensive Analysis	- Analyzes portfolios with asymmetric or complex return distributions.	- May not provide definitive solutions as it assumes multiple probabilistic scenarios.
Simplicity of Risk Modeling	- Effective for risk calculation based on scenarios or Monte Carlo simulations.	- Difficult to interpret results for users without financial expertise.
Time Requirements	- Scales well for problems of any size given sufficient computational resources.	- High computational costs with a large number of simulations.
Integration with Optimization	- Combines with optimization algorithms (e.g., genetic algorithms or simulated annealing).	- Optimization may lead to overfitting if the model does not generalize data effectively.

For analyzing complex portfolio structures, dependencies, and parameter optimization, genetic algorithms and particle swarm optimization are employed. These methods are effective in scenarios involving discrete parameters and complex distribution functions where traditional approaches encounter challenges. Modern algorithms enhance optimization capabilities, enabling more precise consideration of market characteristics and improving strategy adaptability.

Portfolio optimization under modern conditions requires a combination of mathematical methods and market factor analysis. While the Markowitz model remains a theoretical tool, its limitations drive the development of alternative approaches capable of addressing market uncertainty (Alexander N., Scherer W., Burkett M. (2021)). This model is designed to minimize return variance for a fixed mean return.

The mathematical formulation of the problem is as follows:

$$\sigma_p^2 = w^T \sum w \quad (1)$$

Where:

- w — vector of asset weights,

- Σ — covariance matrix of returns,

- σ_p^2 — portfolio variance, representing risk.

Two portfolio types were developed using the model, as presented in Table 3.

Table 3. Portfolio Types (compiled by the author)

Name	**Description**
Portfolio with the highest Sharpe ratio	Assets are allocated as follows: GMKN — 48.8%, DSKY — 22.6%, NKNC — 25.6%. Expected return is 37.1%, volatility is 20.7%. The Sharpe ratio equals 1.70, reflecting optimal risk-return tradeoff.
Portfolio with minimum volatility	Assets are reallocated to minimize risk while maintaining a specified return level.

This tool enables investors to determine the optimal asset allocation given a predefined risk level (Qi Y., Li X., Zhang S. (2021)).

As an example, consider the experiences of the following companies. The organization MegaFon developed video materials to promote tariff plans. This approach increased conversion rates, improving user engagement efficiency. Personalization technologies highlighted key product advantages, enhancing service perception through the personal account (Massahi M., Mahootchi M., Arshadi Khamseh A. (2020)).

Yandex supports employee initiatives, fostering the development of new platforms. Within this framework, the service Yandex.Travel was created, emphasizing the importance of trusting and implementing internal ideas (Massahi M., Mahootchi M., Arshadi Khamseh A. (2020)).

IBM implemented the QRadar Suite, a system for threat analysis, incident monitoring, and process automation. The product simplified cyberattack data management and expedited incident response.

Small businesses leverage artificial intelligence to automate processes. Tasks such as sales analysis, client data management, and optimization are streamlined using modern algorithms.

InniAccounts automated accounting operations using AI, optimizing labor-intensive processes and increasing operational efficiency.

Global corporations are developing metaverse-based approaches. Virtual platforms provide opportunities for digital client interactions, opening prospects for innovative marketing solutions.

Meta is advancing the metaverse concept, creating a platform for user interaction in virtual spaces, enabling opportunities for digital engagement and marketing.

Nvidia launched the Omniverse platform, which facilitates virtual world design, 3D modeling, and AI development, offering new tools for client interaction in digital formats.

For small businesses in general, AI utilization in accounting operations, marketplace interactions, and process optimization enhances task accuracy and reduces time spent on processing (Massahi M., Mahootchi M., Arshadi Khamseh A. (2020)).

Table 4 describes the limitations and advantages of mathematical models for portfolio structure optimization.

Table 4. Limitations and Advantages of Mathematical Models for Portfolio Structure Optimization (compiled by the author)

Type	Limitations	Advantages
Models Based on Classical Portfolio Theory (e.g., Markowitz Model)	- Assumes normality of return distributions.	- Simplicity and clarity.
	- Assumes constant covariances and correlations between assets.	- Analytical solutions are feasible.
	- Sensitive to errors in input data (expected returns and covariances).	- Optimization considers both risk and return.
Linear Programming Models	- Requires accurate data to establish constraints.	- Flexibility in formulating constraints and objectives.
	- Scalability issues with large asset pools.	- Integrates additional factors (e.g., taxes, transaction costs).
Models with Constraints (e.g., Risk-Based Constraints)	- Cannot account for all real-world market constraints and nuances.	- Accounts for specific risks, such as liquidity constraints or minimum asset weights.
	- Often requires significant computational resources.	- Flexibility in adapting to various conditions and investor needs.
Monte Carlo and Simulation-Based Models	- High computational costs, especially with numerous simulations.	- Accommodates non-stationary and non-normal return distributions.
	- Results depend on the quality of random variable modeling.	- Flexibility in modeling diverse market conditions and scenarios.
Game Theory and Evolutionary Algorithm Models	- Complexity in formulation and solution.	- Captures interactions among various market participants.
	- Dependent on the quality of model parameters and strategies.	- Effective for complex, dynamic market conditions.
Artificial Intelligence and Machine Learning Models	- Requires extensive datasets for training.	- Adapts to changing market conditions.
	- Models are often difficult to interpret and lack transparency.	- Improves forecast and optimization accuracy through training on new data.
Option Pricing-Based Models (e.g., Black-Scholes)	- Limited applicability to assets with nonlinear dynamics or lack of liquidity.	- Theoretical rigor and ability to provide precise estimates.
	- Often assumes "ideal" markets (no transaction costs, etc.).	- Suitable for portfolios containing options or derivatives.

Despite the limitations of the classical Markowitz model, including assumptions of normal return distributions and constant parameters, it remains the foundation for many approaches. Its structured application allows for minimizing risks and achieving a balance between return and volatility.

CONCLUSION

The conducted study highlighted the importance of portfolio optimization methods in enhancing asset management quality under conditions of uncertainty. Despite its limitations, the Markowitz model remains a foundational element of portfolio management theory. Its application establishes a structured mechanism aimed at minimizing risk and achieving a balance between return and volatility. However, assumptions regarding normal return distributions, constant parameters, and linear relationships between assets reduce the model's effectiveness in real market conditions.

Modern alternative approaches, such as stochastic programming, robust optimization, and machine learning methods, provide adaptive solutions for operating in unstable markets. These methods account for a wide range of scenarios, asymmetric risks, and complex constraints, which enhance resilience to market parameter fluctuations. However, the high computational complexity and significant resource requirements limit the applicability of these approaches.

Thus, the study justifies the potential of combining the classical model with modern methods for the further development of portfolio management theory. This combination allows leveraging the strengths of each approach while compensating for their weaknesses, which is particularly crucial amidst the increasing instability of global financial markets.

REFERENCES

1. Deng, X. (2022), Research On Finding Optimal Portfolios Based on Markowitz Mean-Variance Model.Constructing Efficient Frontier and Portfolio with Highest Sharpe Ratio Using 4 Assets in Chinese Market. *BCP Business & Management*, 20, 1142-1153.
2. Zhang, M. (2022), Research and Analysis on Markowitz Model of Portfolio Selection.2022 2nd International Conference on Management Science and Industrial Economy Development (MSIED 2022), 219-227.
3. Kalygin V. A., Slashchinin S. V. (2020), Uncertainty of Efficient Frontier in Portfolio Optimization. Learning and Intelligent Optimization: 14th International Conference, LION 14, Athens, Greece, May 24–28, 2020, Revised Selected Papers 14. – Springer International Publishing, 371-376.
4. Alexander N., Scherer W., Burkett M. (2021), Extending the Markowitz model with dimensionality reduction: Forecasting efficient frontiers. 2021 Systems and Information Engineering Design Symposium (SIEDS), 1-6.
5. Chai T. (2023), The Application of the Markowitz Model for Portfolio Optimization in High-End Companies. Advances in Economics, Management and Political Sciences, 44, 107-115.
6. Yao L. (2023), Application of the Markowitz model and indexmodel in real stock markets. Youth Business Review, 1(1), 1-13.
7. Qi Y., Li X., Zhang S. (2021), Optimizing 3-objective portfolio selection with equality constraints and analyzing the effect of varying constraints on the efficient sets. Journal of Industrial and Management Optimization, 17 (4), 1531-1556.
8. Chen, Y., Lu, H., & Yu, T. (2023), Application of the Markowitz model in Construction of Optimal Portfolio in the US stock market. *BCP Business & Management*, 38, 2995-3001
9. Wang, Y. (2023), Select the Optimal Portfolio by Analyzing and Comparing the Better Performance of Markowitz Model and Index Model under 5 Different Constraints. *Advances in Economics, Management and Political Sciences*,13, 364-376.
10. Kim Y. S. (2020), Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk.
11. Massahi M., Mahootchi M., Arshadi Khamseh A.(2020), Development of an efficient cluster-based portfolio optimization model under realistic market conditions. Empirical Economics,59 (5), 2423-2442.
12. Monticeli A. R. et al. (2023), Mixture Design of Experiments as Strategy for Portfolio Optimization. Acta Scientiarum. Technology,45, 63500.

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