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Multiple Contracts: The Case the of System of Mixed Amortization

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ABSTRACT

Aiming to conciliate the pros and cons of the system of amortization with constant installments and the system of constant amortization, the Brazilian Housing Financing System (SFH), introduced the so-called System of Mixed Amortization. Besides comparing these three distinct systems of amortization in the case of individual contracts, the paper presents also a comparison of the correspondent multiple contracts versions.

KEYWORDS: *Systems of Amortization. Mixed Amortization. Multiple Contracts.*

INTRODUCTION

Instituted in 1964, the Brazilian House Financing System ("Sistema Financeiro de Habitação - SFH") established that the system of amortization for house financing would be the so-called system of constant installments. Also named the French system of amortization and,more popularly, also known as "Tabela Price-TP" (in honor of Richard Price, 1771).

Specifically, considering a loan in the amount *F*, at the periodic rate *i* of compound interest, with a term of *n* periods, it was determined that the debt would have to be repaid by *n* periodic constant installments, with value *p*, such that; cf de Faro and Lachtermacher (2012):

$$p = i \times F / \left\{1-(1+i)^{-n}\right\} \quad (1)$$

Later, in 1971, the system of constant installments was substituted by the system of constant amortization, known, in Brazil, as "SAC" and, in Italy, as "ammortamentoitaliano"; cf. Marcelli (2019).

According to this system, the sequence of periodic payments follows an arithmetic progression. With first payment

$$p_1 = F \times (i + 1/n) \quad (2)$$

and ratio

$$R = -i \times F / n \quad (3)$$

cf. de Faro and Lachtermacher (2012).

Observing that $p_1 > p$, if $n > 1$, which could result in a decrease in the demand for house financing, the SFH devised, in 1979, in what can be a considered as a Solomonic solution, the so-called Mixed Amortization System ("Sistema Misto de Amortização – SAM").

As a consequence, and that is the way this paper is going to proceed, one can imagine that the value *F* of the loan can be interpreted as being divided into two halves, $F_1 = F/2$ and $F_2 = F/2$. With F_1 being amortized by the system of constant installments. While F_2 being amortized in accordance with the system of constant amortization.

THE CASE OF A SINGLE CONTRACT

As a simple numerical illustration, consider a loan $F = \$100,000.00$, at the periodic rate $i = 1\%$ of compound interest, with a term *n* of 12 periods.

Considering that we will have $F_1 = \$\,50,000.00$, it follows that the 12 constant installments will have the value $p = \$4,442.44$.

Denoting by I_k the k^{th} parcel of interest, by A_k the k^{th} parcel of amortization, by P_k the k^{th} payment, by $S_k = (1+i) \times S_{k-1} - P_k$ the outstanding debt at time *k*, for *k*=1,2,...,*n*, with $S_0 = F_1$, Table 1 presents the evolution of the first half of the full amount.

Table 1. Evolution of F_1 – Constant Installments

k	I_k	A_k	P_k	S_k
0	-	-	-	50,000.00
1	500.00	3,942.44	4,442.44	46,057.56
2	460.58	3,981.86	4,442.44	42,075.70
3	420.76	4,021.68	4,442.44	38,054.01

4	380.54	4,061.90	4,442.44	33,992.11
5	339.92	4,102.52	4,442.44	29,889.60
6	298.90	4,143.54	4,442.44	25,746.05
7	257.46	4,184.98	4,442.44	21,561.07
8	215.61	4,226.83	4,442.44	17,334.25
9	173.34	4,269.10	4,442.44	13,065.15
10	130.65	4,311.79	4,442.44	8,753.36
11	87.53	4,354.91	4,442.44	4,398.45
12	43.98	4,398.45	4,442.44	0.00
∑	3,309.27	50,000.00	53,309.27	-

On the other hand, considering the second half of the full amount F, Table 2 presents the corresponding evolution of the debt F_2=$50,000.00 using the constant amortization method and the same notations.

Table 2. Evolution of F_2 – Constant Amortizations

k	I_k	A_k	P_k	S_k
0	-	-	-	50,000.00
1	500.00	4,166.67	4,666.67	45,833.33
2	458.33	4,166.67	4,625.00	41,666.67
3	416.67	4,166.67	4,583.33	37,500.00
4	375.00	4,166.67	4,541.67	33,333.33
5	333.33	4,166.67	4,500.00	29,166.67
6	291.67	4,166.67	4,458.33	25,000.00
7	250.00	4,166.67	4,416.67	20,833.33
8	208.33	4,166.67	4,375.00	16,666.67
9	166.67	4,166.67	4,333.33	12,500.00
10	125.00	4,166.67	4,291.67	8,333.33
11	83.33	4,166.67	4,250.00	4,166.67
12	41.67	4,166.67	4,208.33	0.00
∑	3,250.00	50,000.00	53,250.00	-

Consolidating the results of Tables 1 and 2, Table 3 presents the evolution of the full debt $F = \$100,000.00$.

Table 3. Evolution of F – Mixed Amortization System

k	I_k	A_k	P_k	S_k
0	-	-	-	100,000.00
1	1,000.00	8,109.11	9,109.11	91,890.89
2	918.91	8,148.53	9,067.44	83,742.36
3	837.42	8,188.35	9,025.77	75,554.01
4	755.54	8,228.57	8,984.11	67,325.45
5	673.25	8,269.18	8,942.44	59,056.26
6	590.56	8,310.21	8,900.77	50,746.05
7	507.46	8,351.65	8,859.11	42,394.41
8	423.94	8,393.50	8,817.44	34,000.91
9	340.01	8,435.76	8,775.77	25,565.15
10	255.65	8,478.45	8,734.11	17,086.69
11	170.87	8,521.57	8,692.44	8,565.12
12	85.65	8,565.12	8,650.77	0.00
∑	6,559.27	100,000.00	106,559.27	-

It should be noted that, in the case of the system of mixed amortization, the sequence of periodic payments also follows an arithmetic progression, with ratio R' given by:

$$R' = i \times F / (2 \times n) \quad (4)$$

With the first payment equal to:

$$P_1^* = (F/2) \times \{ i / [1-(1+i)^{-n}] + i + 1/n \} \quad (5)$$

THE CASE OF MULTIPLE CONTRACTS

Following the methodology proposed in De-Losso et al. (2013), suppose that the financing institution providing the loan decides to substitute a single contract by n individual contracts. One for each one of the n payments of the single contract.

With the loan of the k^{th} individual contract denoted as F_k, being equal to the present value, at the same rate i of the single contract, of the k^{th} payment of the single contract. That is:

$$F_k = P_k / (1+i)^k, \quad k = 1,2,\ldots,n \quad (6)$$

and the amortization component of the k^{th} individual payment, denoted as $\overline{A}_k$, will be:

$$\overline{A}_k = P_k / (1+i)^k, \quad k = 1,2,\ldots,n \quad (7)$$

That is, we will have $\overline{A}_k = F_k$, for $k = 1,2,\ldots,n$. With the unique installment of the k-th multiple contracts, denoted as $\overline{P}_k$, being equal to the k-th payment of the single contract. That is, $\overline{P}_k = P_k$, for $k = 1,2,\ldots,n$.

On the other hand, and this is the justification for substituting a single contract by n individual contracts, the k^{th} interest component, denoted as $\overline{I}_k$, will be:

$$\overline{I}_k = P_k \times \{1-(1+i)^{-k}\}, \quad k = 1,2,\ldots,n \quad (8)$$

From a strict accounting point of view, not taking into consideration the costs of bookkeeping and of registration of each sub-contract, the total of interest is the same both in the case of a single contract and in the case of multiple contracts. That is:

$$\sum_{k=1}^{n} I_k = \sum_{k=1}^{n} \overline{I}_k \quad (9)$$

Table 4, starting with the sequence of the loans of each subcontract, presents the values of the sequence of payments P_k, the sequence of the parcels of interest I_k, in the case of the corresponding single contract, and the sequence $\overline{I}_k$ of the parcels of interest in the case of the adoption of the option of multiple contracts. Additionally, Table 4 presents also the sequence of differences d_k, and the sequence of accumulated values of d_k, denoted as Δ_k, respectively given by:

$$d_k = I_k - \overline{I}_k \quad ,k = 1,2,\ldots,n \quad (10)$$

and

$$\Delta_k = \sum_{\ell=1}^{k} d_\ell \quad (11)$$

Table 4. Comparison of the Single and Multiple Contracts - Mixed Amortization System

k	$F_k = \overline{A}_k$	$\overline{I}_k$	$\overline{P}_k = P_k$	I_k	$d_k = I_k - \overline{I}_k$	Δ_k
1	9,018.92	90.19	9,109.11	1,000,00	909.81	909.81
2	8,888.78	178.66	9,067.44	918.91	740.24	1,650.06
3	8,760.33	265.45	9,025.77	837.42	571.98	2,222.03
4	8,633.55	350.56	8,984.11	755.54	404.98	2,627.02
5	8,508.42	434.02	8,942.44	673.25	239.24	2,866.26
6	8,384.93	515.84	8,900.77	590.56	74.72	2,940.98
7	8,263.05	596.06	8,859.11	507.46	-88.60	2,852.38
8	8,142.76	674.68	8,817.44	423.94	-250.74	2,601.64
9	8,024.04	751.73	8,775.77	340.01	-411.73	2,189.92
10	7,906.87	827.23	8,734.11	255.65	-571.58	1,618.33
11	7,791.24	901.20	8,692.44	170.87	-730.33	888.00
12	7,677.12	973.65	8,650.77	85.65	-888.00	0.00
$\sum$	100,000.00	6,559.27	106,559.27	6,559.27	0.00	

It is interesting to note that the sequence of the values of d_k does not have more than one change of sign and, adopting the proposition in Nordstrom (1972), as the sequence of accumulated values of d_k does not change sign, it follows that d_k has a unique internal rate of return. Which, in this case, is zero.

However, a more relevant comparison must take into consideration the cost of capital of the financial institution providing the loan. Denoting ρ as the periodic rate that identifies the financial institution cost of capital, one must compare the value of $V_s(\rho)$ with the value of $V_m(\rho)$, respectively the present values of the sequences of interest of the single and multiple contracts, given by:

$$V_s(\rho) = \sum_{k=1}^{n} I_k \times (1+\rho)^{-k} \quad (12)$$

and

$$V_m(\rho) = \sum_{k=1}^{n} \overline{I}_k \times (1+\rho)^{-k} \tag{13}$$

Table 5 presents $V_s(\rho)$ and $V_m(\rho)$ for several values of the cost of capital, for the case of our numerical example. Assuming that i and ρ are monthly rates. With ρ_a denoting the cost of capital in annual terms.

Table 5. Present values of interest sequences. – Mixed Amortization Case

ρ_a	ρ	$V_s(\rho)$	$V_m(\rho)$	%(difference)
5%	0.40741%	12,657.47	12,564.93	0.73647978
10%	0.79741%	12,237.04	12,060.73	1.46187785
15%	1.17149%	11,852.01	11,599.53	2.17658559
20%	1.53095%	11,497.98	11,176.00	2.88097576
25%	1.87693%	11,171.28	10,785.65	3.57540286
30%	2.21045%	10,868.78	10,424.66	4.26020378

Therefore, in the case of our simple numerical example, we have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$. That is, the financial institution providing the loan should prefer to implement the multiple contracts option. Furthermore, considering different values of the financing rate i, as well as distinct values of the term n of the contract, it may be shown that we always have $V_s(\rho) > V_m(\rho)$ if $\rho > 0$.

GENERAL ANALYSIS

Defining as the fiscal gain, which will be denoted by $\delta$and given by the following expression, in percentage

$$\delta(\%) = 100 \times \{V_s(\rho)/V_m(\rho) - 1\} \tag{14}$$

Tables 6 to 8 present the values of the fiscal gain δ(%),when the financing interest rate i varies from 0.5% monthly to 2% monthly, and the term n of the contract goes from 5 to 30 years. With the annual value of the financial institution cost of capital ranging from 5% to 30%.

Table 6. Fiscal Gains δ(%) – Mixed Amortization Method – i=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.6825	15.5168	23.4761	31.5349	39.6694	47.8572
10	14.8685	30.7903	47.5872	65.0710	83.0556	101.3670
15	21.3873	45.0337	70.3895	96.8771	123.9663	151.2181
20	27.2393	57.9093	90.8068	124.7845	158.9431	192.6764
25	32.4511	69.2677	108.3712	148.0569	187.2419	225.3663
30	37.0662	79.1121	123.1035	166.9707	209.6860	250.8725

Table 7. Fiscal Gains δ(%)– Mixed Amortization Method – i=1.0%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.1227	14.3365	21.6170	28.9421	36.2911	43.6457
10	12.8714	26.3602	40.3073	54.5600	68.9799	83.4467
15	17.3961	35.9156	55.1307	74.6588	94.1925	113.5060
20	20.9617	43.3693	66.4382	89.5578	112.3234	134.5030
25	23.7884	49.1477	74.9330	100.3992	125.1589	149.0551
30	26.0491	53.6374	81.3221	108.3384	134.3905	159.4140

Table 8. Fiscal Gains δ(%)– Mixed Amortization Method – i=2%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.1916	12.3941	18.5893	24.7613	30.8961	36.9818
10	10.0435	20.2786	30.5991	40.9144	51.1509	61.2514

15	12.4877	25.2493	38.0549	50.7312	63.1598	75.2674
20	14.1168	28.4988	42.7979	56.7938	70.3689	83.4735
25	15.2546	30.7123	45.9355	60.7001	74.9199	88.5814
30	16.0821	32.2791	48.0986	63.3425	77.9656	91.9846

As is shown in Tables 6 to 8, the values of $\delta$are very significant and always positive. Indicating that the best option for the financial institution is to implement multiple contracts.

As a further illustration, if the rate i is 0.5%, 1% or 2% per month, Figures 1,2 and3 depict the behavior of$\delta$when the opportunity cost of the financial institution varies from to 5% to 30% in annual terms, and the term of the contract on annual terms varies from 5 to 30 years.

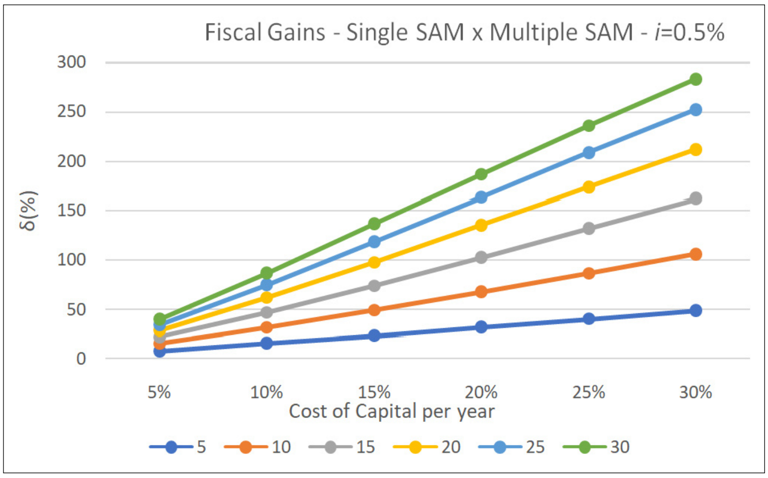

Figure 1. Fiscal Gains δ(%) – *i* = 0.5% per month

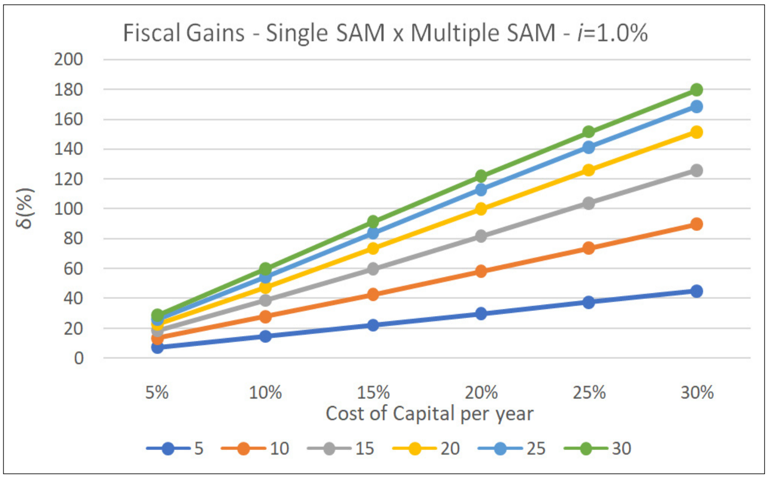

Figure 2. Fiscal Gains δ(%) – *i* = 1.0% per month

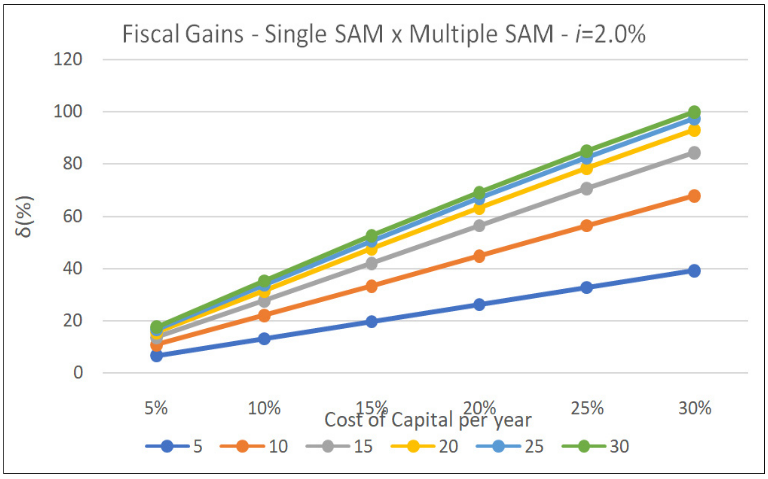

Figure 3. Fiscal Gains δ(%) – *i* = 2.0% per month

COMPARISON OF MIXED AMORTIZATION, CONSTANT INSTALLMENTS AND CONSTANT AMORTIZATION

Given that the financing institution could either offer the constant installments or the constant amortization or the mixed methods, and since all the corresponding multiple contracts versions offer significant fiscal gains over the corresponding single contracts, the financial institution should decide which method gives the best result, considering the corresponding interest sequences and its own cost of capital.

If the loan values are the same, and the cost of capital is not considered, the method which offers the bigger total interest payments should be chosen. On the other hand, considering the financial institution's cost of capital, the percentage difference of the present values of the corresponding interest sequences of the methods should be the criteria of decision

Constant installments (TP) versus constant amortization (SAC)

If the loan values are the same, and the cost of capital is not considered, the method which offers the bigger total interest payments should be chosen. Table 9 and Figure 4 present the total difference of interest charged by the method of constant installments (TP) and the method of constant amortization (SAC). For the case where $F = \$100,000.00$.

Table 9. Total Difference of Interest – TP x SAC

	Total Interest Difference		
Term / InterestRate	0.5	1.0	2.0
60	746,81	2.966,69	11.607,79
120	2.974,60	11.665,14	43.577,16
240	11.693,45	43.760,67	89.644,77
360	25.588,19	89.800,53	259.577,59

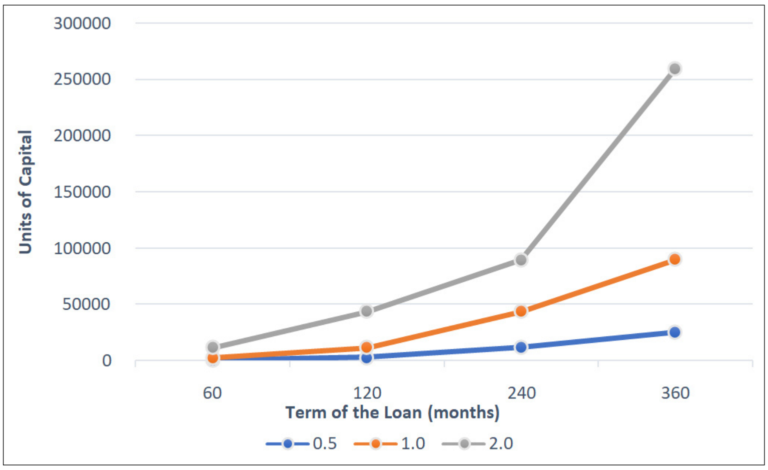

Figure 4. Total Interest Difference – TP x SAC

As shown, the total interest difference increases exponentially with the term and with the interest rate of the loan. So, by this criterion the financial institution should prefer to offer the constant installments method. As also shown in the study of the case of thirteen annual wages that was analyzed in de Faro and Lachtermacher (2025a and b).

Denote $V^{TP}(\rho)$ and $V^{SAC}(\rho)$ the present values of the interest sequences of the constant installments (TP) and of the constant amortization (SAC) methods, respectively given by:

$$V^{TP}(\rho) = \sum_{k=1}^{n} \overline{I}_k^{TP} \times (1+\rho)^{-k} \qquad (15)$$

and

$$V^{SAC}(\rho) = \sum_{k=2}^{n} \overline{I}_k^{SAC} \times (1+\rho)^{-k} \qquad (16)$$

where $\overline{I}_k^{TP}$ and $\overline{I}_k^{SAC}$ are the sequences of interest payments in the case of multiple contracts, considering constant installments and constant amortization.

In this case, percentage difference of the present values of the corresponding interest sequences, denoted a δ^{TP-SAC}, which does not depend on *F*, is given by:

$$\delta^{TP-SAC}(\%) = 100 \times \{V^{TP}(\rho) / V^{SAC}(\rho) - 1\} \qquad (17)$$

Tables 10, 11 and 12, and Figures 5, 6, and 7, respectively relative to cases where the financing rate is 0.5%, 1% and 2% monthly, present the values of the corresponding percentage difference $\delta^{TP-SAC}(\%)$.

Table 10. Percentage Difference $\delta^{TP-SAC}(\%)$– TP x SAC – *i*=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	4,49316	4,10047	3,71947	3,35047	2,99364	2,64905
10	8,27400	6,76488	5,32239	3,95863	2,68143	1,49472
15	11,30639	8,12070	5,20055	2,58682	0,29011	-1,70169
20	13,66080	8,46945	4,00278	0,29532	-2,71259	-5,12579
25	15,42004	8,12909	2,32783	-2,09495	-5,40526	-7,88301
30	16,67043	7,38015	0,60233	-4,14352	-7,45224	-9,80200

Table 11. Percentage Difference $\delta^{TP-SAC}(\%)$ – TP x SAC –*i*=1.0%p.m.

	$\rho_a(\%)$					
n(years)	5%	10%	15%	20%	25%	30%
5	8,9483	8,1978	7,4754	6,7812	6,1149	5,4761
10	16,3875	13,6619	11,1235	8,7816	6,6379	4,6874
15	22,1900	16,7863	12,0544	7,9873	4,5365	1,6307
20	26,4777	18,2081	11,5209	6,2436	2,1262	-1,0824
25	29,4309	18,4867	10,3911	4,5388	0,3134	-2,7782
30	31,2613	18,0541	9,1443	3,2113	-0,8080	-3,6216

Table 12. Percentage Difference $\delta^{TP-SAC}(\%)$ – TP x SAC – *i*=2%p.m.

	$\rho_a(\%)$					
n(years)	5%	10%	15%	20%	25%	30%
5	17,5791	16,1998	14,8893	13,6455	12,4659	11,3479
10	30,9793	26,4097	22,2982	18,6237	15,3556	12,4582
15	39,6520	31,3675	24,4792	18,8147	14,1817	10,3950
20	44,3883	32,6839	23,7904	17,1035	12,0705	8,2475
25	46,3233	31,8534	21,8441	14,9411	10,1075	6,6357
30	46,4541	29,9511	19,5515	12,9172	8,5298	5,4944

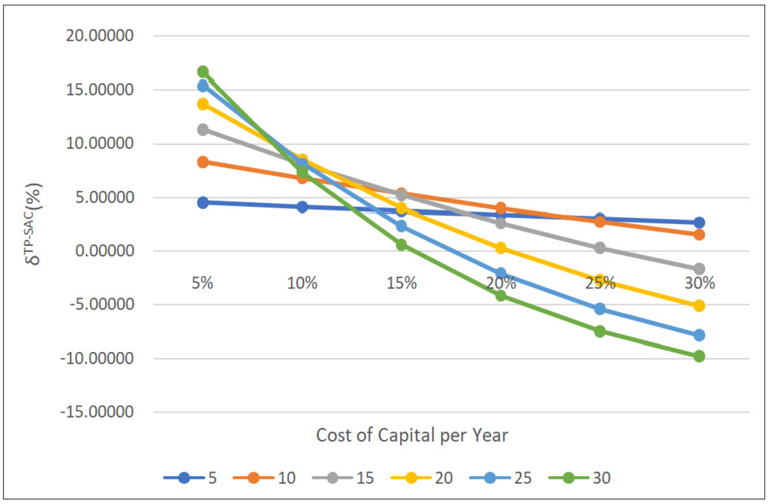

Figure 5. $\delta^{TP-SAC}(\%)$– TP x SAC – *i*=0.5%p.m.

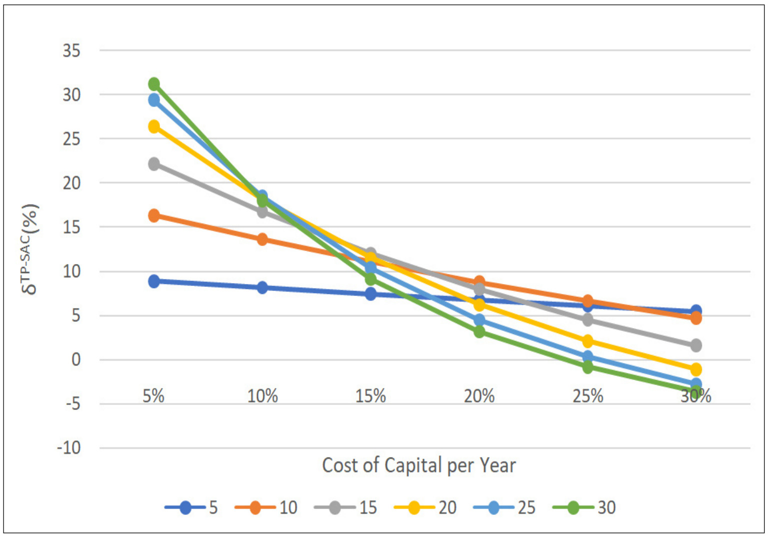

Figure 6. $\delta^{TP-SAC}(\%)$ – TP x SAC – *i*=1.0%p.m.

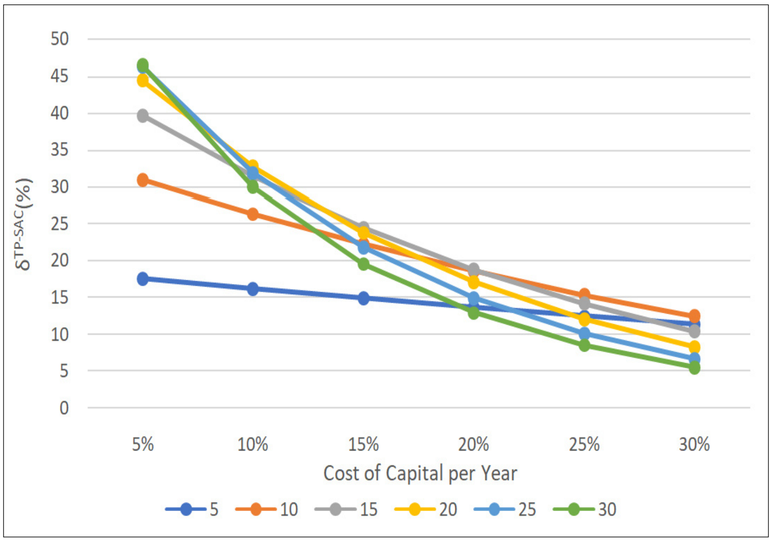

Figure 7. $\delta^{TP-SAC}(\%)$ – TP x SAC – *i*=1.0%p.m.

As shown in Tables 10, 11 and 12, and Figures 5, 6, and 7, most of the present values of the constant installments (TP) interest sequences are larger than the constant amortization (SAC) ones. However, increasing the cost of capital and the term of the loan, especially for low interest rates, we will have cases with bigger present values for the constant amortization method.

Therefore, confirming de Faro and Lachtermacher (2025a and b), if the criteria for choosing the best option is the corresponding present value of the interest sequence, the choice will depend on the interest rate, on the term of the loan, as well as on the cost of capital of the financial institution.

Constant installments (TP) versus Mixed amortization (SAM)

If the loan values are the same, and the cost of capital is not considered, the method which offers the bigger total interest payments should be chosen. Table 13 and Figure 8 present the total difference of interest charged by the method of constant installments (TP) and the method of mixed amortization (SAM). For the case where $F = \$100,000.00$.

Table 13. Total Difference of Interest – TP x SAM

	Total Interest Difference		
Term / InterestRate	0.5	1.0	2.0
60	373,40	1.483,34	5.803,90
120	1.487,30	5.832,57	21.788,58
240	5.846,73	21.880,34	71.588,98
360	12.794,09	44.900,27	129.788,79

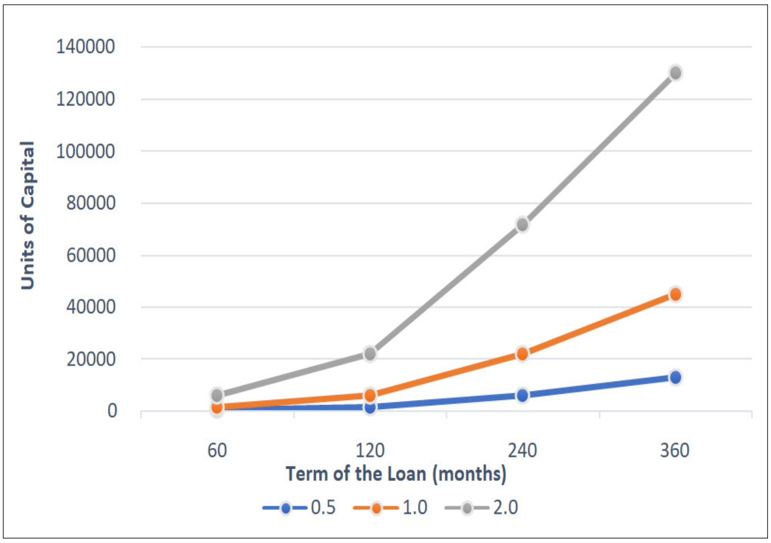

Figure 8. Total Interest Difference – TP x SAM

As shown, the total interest difference increases exponentially with the term and with the interest rate of the loan. So, according to this criterion the financial institution should prefer to offer the constant installments method.

Considering the cost of capital of the financial institution, the criterion for choosing the best model should be the percentage difference of the corresponding present values of the interest sequences. Denoting $V^{SAM}(\rho)$ the present value of the interest sequence of the mixed amortization (SAM) method is:

$$V^{SAM}(\rho)=\sum_{k=1}^{n} \overline{I}_k^{SAM} \times (1+\rho)^{-k} \qquad (18)$$

And the percentage difference of the present values of the corresponding interest sequences, denoted a δ^{TP-SAM}, is given by:

$$\delta^{TP-SAM}(\%)=100\times\left\{V^{TP}(\rho)/V^{SAM}(\rho)-1\right\} \qquad (19)$$

Tables 14, 15 and 16, and Figures 9, 10, and 11, respectively relative to cases where the financing rate is 0.5%, 1% and 2% monthly, present the values of the corresponding percentage difference $\delta^{TP-SAM}(\%)$.

Table 14. Percentage Difference $\delta^{TP-SAM}(\%)$ – TP x SAM – *i*=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	2,1972	2,0090	1,8258	1,6476	1,4747	1,3072
10	3,9727	3,2718	2,5922	1,9409	1,3230	0,7418
15	5,3507	3,9019	2,5344	1,2769	0,1448	-0,8581
20	6,3937	4,0627	1,9621	0,1474	-1,3749	-2,6303
25	7,1581	3,9058	1,1505	-1,0586	-2,7777	-4,1032
30	7,6939	3,5588	0,3003	-2,1156	-3,8703	-5,1536

Table 15. Percentage Difference $\delta^{TP-SAM}(\%)$ – TP x SAM – *i*=1.0%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	4,2826	3,9375	3,6030	3,2794	2,9667	2,6651
10	7,5732	6,3942	5,2687	4,2061	3,2123	2,2900
15	9,9869	7,7433	5,6846	3,8403	2,2179	0,8088
20	11,6911	8,3444	5,4467	3,0273	1,0519	-0,5442
25	12,8278	8,4613	4,9389	2,2190	0,1564	-1,4087
30	13,5177	8,2796	4,3722	1,5803	-0,4057	-1,8442

Table 16. Percentage Difference $\delta^{TP-SAM}(\%)$ – TP x SAM – *i*=2%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	8,0794	7,4930	6,9288	6,3870	5,8673	5,3693
10	13,4122	11,6646	10,0308	8,5186	7,1303	5,8638
15	16,5456	13,5575	10,9049	8,5985	6,6213	4,9407
20	18,1630	14,0465	10,6307	7,8780	5,6918	3,9604
25	18,8059	13,7386	9,8466	6,9513	4,8106	3,2113
30	18,8490	13,0250	8,9052	6,0668	4,0905	2,6738

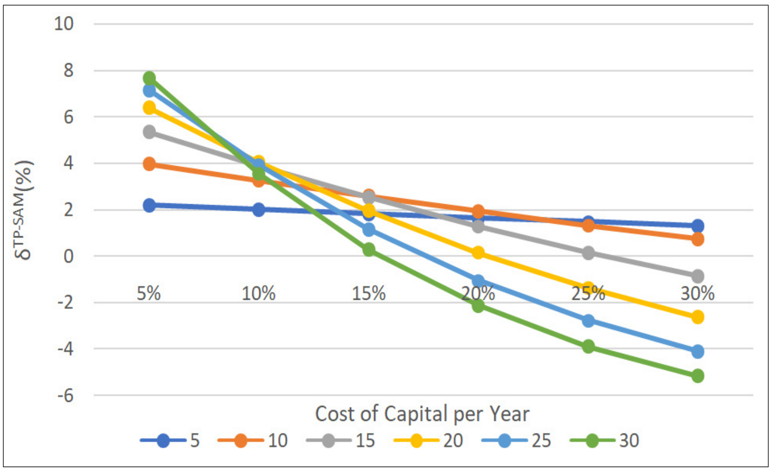

Figure 9. $\delta^{TP-SAM}(\%)$ – TP x SAM – *i*=0.5%p.m.

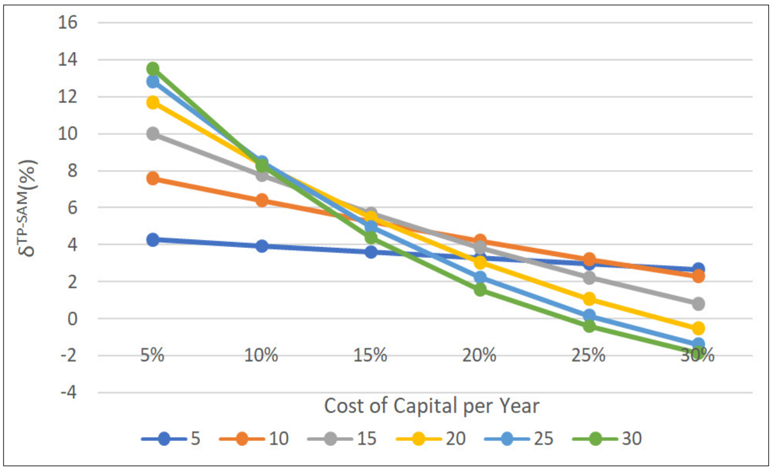

Figure 10. $\delta^{TP-SAM}(\%)$ – TP x SAM – *i*=1.0%p.m.

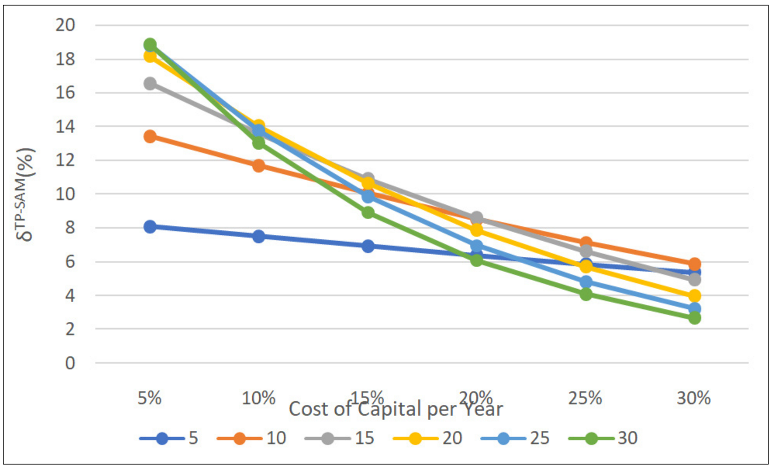

Figure 11. $\delta^{TP-SAM}(\%)$ – TP x SAM – *i*=1.0%p.m.

As shown in Tables 14, 15 and 16, and Figures 9, 10, and 11, most of the present values of the constant installments (TP) interest sequences are larger than the mixed amortization (SAM) ones. However, increasing the cost of capital and the term of the loan, especially for low interest rates, we will have cases with bigger present values for the mixed amortization method.

Therefore, if the criterion for choosing the best option is the corresponding present value of the interest sequence, the choice will depend on the interest rate, on the term of the loan, as well as on the cost of capital of the financial institution.

Mixedamortization (SAM) versus Constant amortization (SAC)

If the loan values are the same, and the cost of capital is not considered, the method which offers the bigger total interest payments should be chosen. Table 17 and Figure 12 present the total difference of interest charged by the mixed amortization method (SAM) and the constant amortization method (SAC). For the case where $F = \$100,000.00$.

Table 17. Total Difference of Interest – SAM x SAC

	Total Interest Difference		
Term / InterestRate	0.5	1.0	2.0
60	373,40	1.483,34	5.803,90
120	1.487,30	5.832,57	21.788,58
240	5.846,73	21.880,34	71.588,98
360	12.794,09	44.900,27	129.788,79

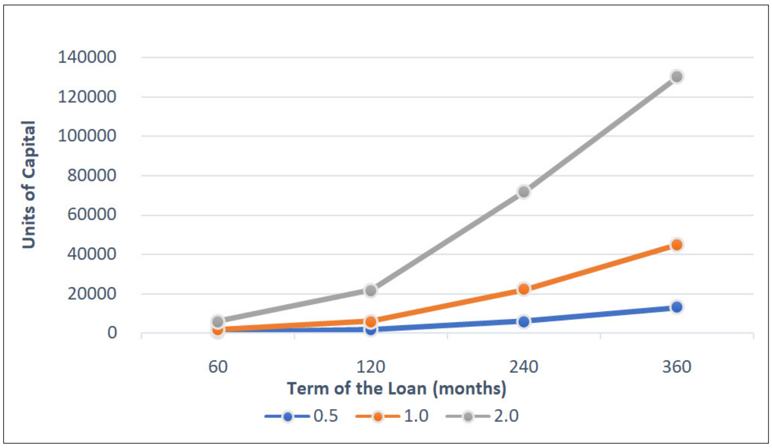

Figure 12. Total Interest Difference – SAM x SAC

As shown, the total interest difference increases exponentially with the term and with the interest rate of the loan. So, according to this criterion the financial institution should prefer the mixed amortization method.

It should be noted that Tables 13 and 17 and Figures 8 and 12 are equal. The reason for this to happens is connected to the fact that the mixed amortization method is based on the mixture of the constant installments (TP-50%) and constant amortization (SAC-50%) and the fact that the differences were based on the TP – SAM and SAM – SAC.

Considering the percentage difference of the present values of the corresponding interest sequences, denoted a $\delta^{SAM-SAC}$, is given by:

$$\delta^{SAM-SAC}(\%) = 100 \times \left\{ V^{SAM}(\rho) / V^{SAC}(\rho) - 1 \right\} \quad (20)$$

Tables 18, 19 and 20, and Figures 13, 14, and 15, respectively relative to cases where the financing rate is 0.5%, 1% and 2% monthly, present the values of the corresponding percentage difference $\delta^{SAM-SAC}(\%)$.

Table 18. Percentage Difference $\delta^{SAM-SAC}(\%)$ – SAM x SAC – *i*=0.5%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	2,2466	2,0502	1,8597	1,6752	1,4968	1,3245
10	4,1370	3,3824	2,6612	1,9793	1,3407	0,7474
15	5,6532	4,0604	2,6003	1,2934	0,1451	-0,8508
20	6,8304	4,2347	2,0014	0,1477	-1,3563	-2,5629
25	7,7100	4,0645	1,1639	-1,0475	-2,7026	-3,9415
30	8,3352	3,6901	0,3012	-2,0718	-3,7261	-4,9010

Table 19. Percentage Difference $\delta^{SAM-SAC}(\%)$ – SAM x SAC – *i*=1.0%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	4,4742	4,0989	3,7377	3,3906	3,0574	2,7381
10	8,1938	6,8310	5,5617	4,3908	3,3189	2,3437
15	11,0950	8,3932	6,0272	3,9937	2,2682	0,8153
20	13,2389	9,1040	5,7605	3,1218	1,0631	-0,5412
25	14,7155	9,2434	5,1955	2,2694	0,1567	-1,3891
30	15,6306	9,0270	4,5721	1,6057	-0,4040	-1,8108

Table 20. Percentage Difference $\delta^{SAM-SAC}(\%)$ – SAM x SAC – *i*=2%p.m.

	ρa(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	8,7896	8,0999	7,4446	6,8228	6,2330	5,6740
10	15,4897	13,2048	11,1491	9,3119	7,6778	6,2291
15	19,8260	15,6838	12,2396	9,4074	7,0908	5,1975
20	22,1942	16,3420	11,8952	8,5518	6,0353	4,1237
25	23,1616	15,9267	10,9220	7,4706	5,0538	3,3179
30	23,2271	14,9755	9,7757	6,4586	4,2649	2,7472

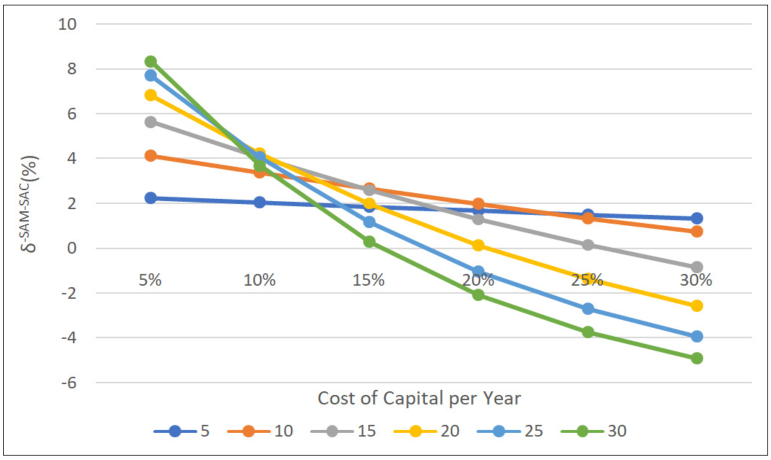

Figure 13. $\delta^{SAM-SAC}(\%)$ – SAM x SAC – *i*=0.5%p.m.

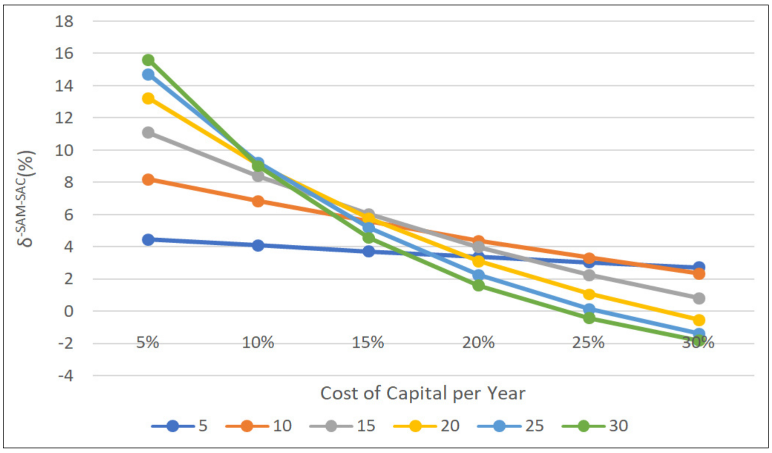

Figure 14. $\delta^{SAM-SAC}(\%)$ – SAM x SAC – *i*=1.0%p.m.

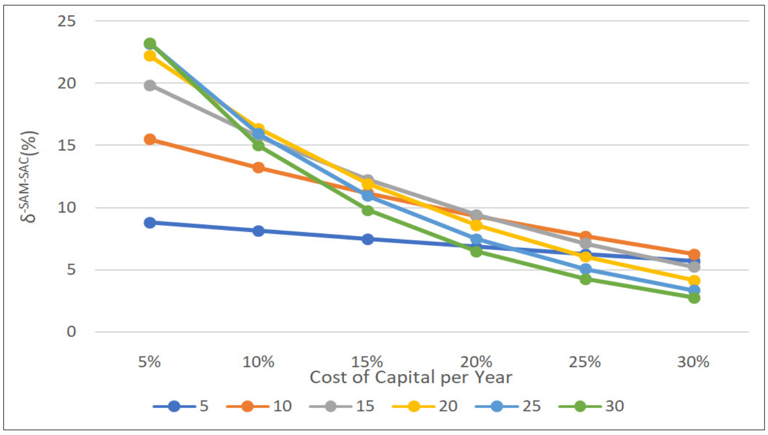

Figure 15. $\delta^{SAM-SAC}(\%)$ – SAM x SAC – *i*=2.0%p.m.

As shown in Tables 18, 19 and 20, and Figures 13, 14, and 15, most of the present values of the mixed amortizations (SAM) interest sequences are larger than the constant amortization (SAC) ones. However, increasing the cost of capital and the term of the loan, especially for low interest rates, we will have cases with bigger present values for the mixed amortization method.

Therefore, if the criterion for choosing the best option method is the present value of the interest sequence, the choice will depend on the interest rate, on the term of the loan, as well as on the cost of capital of the financial institution.

CONCLUSIONS

In this paper we have considered the so-called system of mixed amortization, which was established in 1979by the Brazilian System of House Financing.

Focusing attention in the case where a single contract is substituted by multiple contracts, it was shown that the best option for the financing institution providing the loan is to implement the multiple contracts version.

However, taking into account that the financing institution providing the loan may also implement either the constant installment method or the system of constant amortization, it was shown that the best option will depend on the interest rate being charged, on the term of the loan, as well as on the cost of capital of the financing institution.

A result that extends the previous contributions of De-Losso et al. (2013), de Faro (2022), de Faro and Lachtermacher (2025a) and de Faro and Lachtermacher (2025b).

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