American Research Journal of Business and Management

Volume 7, Issue 1, 1-6 Pages
Research Article | Open Access
ISSN (Online)- 2379-1047
DOI :

AMERICAN RESEARCH JOURNALS
An Academic Publishing House

Multiple Contracts: The Case of Periodic Payments Only

Clovis de Faro

Graduate School of Economics, EPGE-FGV, Rio de Janeiro, Brazil.

ABSTRACT

As previously observed by De-Losso et al (2013), for the case of a loan that is to be repaid by constant payments, and also by the Faro (2017), for the case of the adoption of the constant amortization scheme, the pratice of substituting a simple contract by n subcontracts, may also result in a substantial fiscal gain for the financing institution. In the present paper the analysis will be extended to address the case where the borrower is required to pay interest only, with the principal being repaid by a lump-sum payment at the end of the loan term.

INTRODUCTION

Motivated by the controversy regarding the occurrence of anatocism, which is defined as repeated or doubled interest (cf. TheLaw.com Dictionary), in the constant payments scheme of debt amortization, Sandrini (2007, p. 127) should be credited as the originator of the idea of substituting a single contract by *n* subcontracts: one for each of the *n* periodic payments.

Focusing attention on the traditional constant payments scheme, De-Losso et al (2013) has shown that substituting a single contract by subcontracts may result, in terms of present values, in substantial income tax reductions for the financing institution.

For the constant amortization scheme, which is very popular for home financing in Brazil, de Faro (2017), has also shown that the financial institution granting the loan may derive even greater income tax reductions, if the practice of multiple contracts is adopted.

The focus of the present paper is to extend the analysis to the case where the borrower is required to periodically pay interest only, with the principal of the loan being repaid by a lump-sum payment at the end of the *n*-th period of the loan term.

Noting that this particular amortization scheme is known in Brazil as "métodoamericano" (cf. Puccini (1999) and de Faro e Lachtermacher (2012)), it should be pointed out as mentioned in Butcher and Nesbitt (1971) and in Kellison (1991), that the borrower is usually required to establish what is called a sinking-fund in order to face the lump-sum payment at the end of the loan.

THE "MÉTODO AMERICANO" IN THE CASE OF A SINGLE CONTRACT

Consider a loan amount F, that has to be repaid at the periodic interest rate i, with the term of *n* periods.

In the event of a single contract, in accordance to the so called "sistema americano", it is well known (cf. de Faro and Lachtermacher, 2012, p.260) that the *k*-th payments, denoted by p_k, is equal to:

$$p_k = \begin{cases} i.F, & k = 1, 2, \dots, n-1 \\ F(1+i), & k = n \end{cases} \tag{1}$$

The financial interpretation ensues that the periodic interest payment, at the rate i, against principal F is paid, with the principal being repaid, jointly with the last parcel of interest at the end of the term *n*.

Denoting by A_k and J_k, respectively, the *k*-th parcels of amortization and of interest, we have that:

$$A_k = \begin{cases} 0, \text{ for } k = 1, 2, \dots, n-1 \\ F, \text{for } k = n \end{cases} \tag{2}$$

and

$$J_k = i.F, \quad \text{for } k = 1, 2, \dots, n \tag{3}$$

with the total of accounting interest, denoted as $\bar{J}$, being equal to:

$$\bar{J} = \sum_{k=1}^{n} J_k = n.i.F \tag{4}$$

Simple or Compound Interest

Although it should be considered as trivial, there is a subjacent question that can be traced back at least with the publication of the work of Wilkie (1794). Page 7 states that:

"When a sum of money is lent upon interest, and that interest is either kept in the borrower's hand or is paid regularly, without becoming part of the principal, it is then said to bear simple interest".

The above question, concerning the interpretation of the nature of the interest being paid, whether simple or compound, appears in Veras (1991, p. 193) as mentioned in Sandrini (2007, p. 61), and also in Halter (2013, p. 52).

To address the above question, it suffices to state that, contrary to the usual case of a loan taken at simple interest, when the payment of interest occurs only once at the end of the term of *n*periods, simultaneously with the repayment of the principal, in the "métodoamericano" the lender has the opportunity to reinvest each one of the periodic payments in the capital market.

That is, for the case of loan F, at the periodic rate i of simple interest, with a term of *n*periods, we have, at the end of the term, just one total payment equal to $F(1+i.n)$, with the total interest equal to $n.i.F$.

On the other hand, if the loan is taken in accordance with the "sistemaamericano", even though we will have, from an accounting point of view, the same total of $n.i.F$ as interest, we have to consider the possibility of reinvesting each of the parcels of interest.

In particular, if the periodic rate of compound interest prevailing in the capital market at the time of each reinvestment is equal to i', it follows that the lender, at the end of the term of *n* periods, will be able to accumulate the total amount S_n, equal to:

$$S_n = \sum_{k=1}^{n} p_k (1+i')^{n-k} = i.F \sum_{k=1}^{n} (1+i')^{n-k} + F \qquad (5)$$

That is:

$$S_n = i.F\left\{(1+i')^n - 1\right\}/i' + F$$

Thus, taking into account Newton's binominal, for the expansion of $(1+i')^n$, it follows that $S_n > n.i.F + F$, if $i' > 0$ and $n > 1$.

In particular, if $i' = i$, we will have:

$$S_n = F(1+i)^n \qquad (5')$$

Equivalently, it is made evident that the present value of the*n*payments given by (1) at the compound interest rate i is equal to the principal F. That is, we have the classical formula of financial equivalence at the rate i of compound interest between the principal and the sequence of payments:

$$\sum_{k=1}^{n} p_k (1+i)^{-k} = F \qquad (6)$$

Therefore, it can be unequivocally concluded that the considered rate i is, effectively, of compound interest.

THE n SUBCONTRACTS OPTION

Suppose now that instead of a single contract with the principal amount *F*, the financial institution decides to issue *n* subcontracts also considering the periodic interest rate i; *i.e.* one subcontract for each one of the *n* payments of the single contract, with the principal amount of the *k*-th subcontract, denoted as F_k, being equal to the present value at the rate i of the *k*-th payment p_k of the single contract.

That is, the principal amount of the *k*-th subcontract is:

$$F_k = p_k (1+i)^{-k} = \begin{cases} i.F(1+i)^{-k}, & \text{for } k = 1, 2\ldots, n-1 \\ F(1+i)(1+i)^{-n} = F(1+i)^{1-n}, & \text{for } k = n \end{cases} \qquad (7)$$

In this case, the parcels of amortization and accounting interest, respectively denoted as $\hat{A}_k$ and $\hat{J}_k$, associated with the *k*-th subcontract, are:

$$\hat{A}_k = F_k = \begin{cases} i.F(1+i)^{-k}, & \text{for } k = 1, 2, \ldots, n-1 \\ F(1+i)^{1-n}, & \text{for } k = n \end{cases} \qquad (8)$$

and

$$\hat{J}_k = p_k - \hat{A}_k = \begin{cases} i.F\left\{1-(1+i)^{-k}\right\}, & \text{for } k = 1, 2, \ldots, n-1 \\ F(1+i)\left\{1-(1+i)^{-n}\right\}, & \text{for } k = n \end{cases} \qquad (9)$$

with the total accounting of interest for the *n* subcontracts, which will be denoted as $\hat{J}$, being equal to:

$$\hat{J} = \sum_{k=1}^{n} \hat{J}_k = i.F \sum_{k=1}^{n} \left\{1-(1+i)^{-k}\right\} + F\left\{1-(1+i)^{-n}\right\}$$

or

$$\hat{J}_k = i.F\left\{n - \sum_{k=1}^{n} (1+i)^{-k}\right\} + F - F(1+i)^{-n}$$

$$= i.F\left\{n - \left[1-(1+i)^{-n}\right]/i\right\} + F - F(1+i)^{-n}$$

Therefore, exactly as in the case of a single contract, the total of accounting interest for the case of the *n* subcontracts, is:

$$\hat{J} = n.i.F \qquad (10)$$

THE FISCAL GAIN

Although the total of accounting interest is the same both in the case of a single contract and in the case of *n* subcontracts, one has to take into consideration the financial institution's opportunity cost.

Denoting by ρ the interest rate relative to the same period as the financing rate i, which expresses the opportunity cost for the financing institution, we must compare the present value of the sequence of the parcels of interest in the case of a single contract with the present value of the corresponding sequence of parcels of interest in the case of the *n* subcontracts. That is, denoting by $V_1(\rho)$ and by $V_2(\rho)$ the respective present values at the rate ρ, the financial institution will be better off in terms of fiscal gains, if $V_1(\rho) > V_2(\rho)$

A Numerical Example

Before the presentation of a general analysis, it is opportune to present a numerical example.

To this end, consider a loan of $ 1,000,000.00 with a term of 12 months, at the monthly interest rate of 2%.

Table 1 depicts the respective evolutions of $S_k, p_k, J_k, \hat{A}_k = F_k$, and of k, as well of the difference $\delta_\kappa = J_k - \hat{J}_k$, for $k = 1, 2, \ldots, 12$.

Table 1. Evolution of the Numerical Values of the Example

k	S_k	P_k	J_k	$\hat{A}_k = F_k$	$\hat{J}_k$	$\delta_\kappa = J_k - \hat{J}_k$
0	1,000,000.00	-	-	-	-	-
1	"	20,000.00	20,000.00	19,607.84	392.16	19,607.84
2	"	"	"	19,223.38	776.62	19,223.38
3	"	"	"	18,846.45	1,153.55	18,846.45
4	"	"	"	18,476.91	1,523.09	18,476.91
5	"	"	"	18,144.62	1,885.38	18,114.62
6	"	"	"	17,759.43	2,240.58	17,759.42
7	"	"	"	17,411.20	2,588.80	17,411.20
8	"	"	"	17,069.81	2,930.19	17,069.81
9	"	"	"	16,735.11	3,264.89	16,735.11
10	"	"	"	16,406.97	3,593.03	16,406.97
11	"	"	"	16,085.26	3,914.75	16,085.25
12	0.00	1,020,000.00	"	804,263.04	215,736.96	-195,736.96
Σ	_	1,240,000.00	240,000.00	1,000,000.00	240,000.00	0.00

(Values in $)

There are two points in Table 1 that should be pointed out.

The first one, which is trivial, is that in the case of a single contract, all parcels of interest are equal to $ 20,000.00

The second is that in the case of multiple contracts, the parcels of interest yield an increasing sequence. Thus, in the general case of *n* subcontracts, we have:

$$\hat{J}_{k+1} - \hat{J}_k = i^2 F (1+i)^{-k-1} > 0, \quad \text{if } i > 0, \quad \text{for } k = 1, 2, \ldots, n-2 \tag{11}$$

with

$$\hat{J}_n - \hat{J}_{n-1} = F \left\{ 1 - (1+i)^{-n} \right] + i^2 (1+i)^{-n} > 0, \quad \text{if } i > 0 \tag{12}$$

It follows then, as illustrated on the last column of Table 1, that the sequence of differences $\delta_\kappa = J_k - \hat{J}_k$, for $k = 1, 2, \ldots, n$, is analogous to the one which characterizes what is named to be a conventional project, which, as is well known (cf. de Faro 1971 and 1974), has a unique internal rate of return.

Therefore, taking into account that $V_1(0) = V_2(0) = n.i.F$, which means that the considered internal rate of return is null, it follows that $V_1(\rho) > V_2(\rho)$, if $\rho > 0$

The General Case

In the general case of a loan with a term of *n* periods, we have, in the case of a unique contract, that:

$$V_1(\rho) = i.F\, \{1 - (1+\rho)^{-n}\} / \rho \text{ , if } \rho > 0 \tag{13}$$

On the other hand, in the case of *n*subcontracts, we will have that:

$$V_2(\rho) = i.F \left\{ \left[1 - (1+\rho)^{-n} \right] / \rho - \left[1 - (1+\hat{\rho})^{-n} \right] / \hat{\rho} + (1+\rho)^{-n} \left[1 - (1+i)^{-n} \right] / i \right\} \tag{14}$$

where $\hat{\rho} = i + \rho + i.\rho$

However, although we already know that $V_1(\rho) > V_2(\rho)$, if $\rho > 0$, we still have to provide numerical evidence that the difference is significant.

To this end, Table 2 and 3, which respectively refer to the cases where the monthly interest rate i being charged against the loan is 2% and 3%, present the numerical values of what is defined as the fiscal gain $\delta = V_1(\rho)/V_2(\rho) - 1$, for some values of the annual interest rate ρ_a which designates the opportunity cost for the financial institution, for annual loan terms of 1 to 20, and monthly payments as well.

Table 2. Percent Fiscal Gain When $i = 2\%$

Annual term	Annual opportunity cost ρ_a (%)						
	0	5	10	15	20	25	30
1	0	2.1	4.1	6.1	8.1	10.0	11.9
2	0	4.1	8.2	12.3	16.5	25.1	34.3
3	0	5.9	11.8	18.1	24.2	30.6	37.0
4	0	7.4	15.1	23.0	31.2	39.7	45.4
5	0	8.8	17.9	27.6	37.6	47.9	71.9
10	0	13.4	27.8	43.4	59.0	75.3	91.6
20	0	17.3	46.5	54.4	72.7	90.2	106.8

Table 3. Percent Fiscal Gain When $i = 3\%$

Annual term	Annual opportunity cost ρ_a (%)						
	0	5	10	15	20	25	30
1	0	2.0	4.0	5.9	7.8	9.6	11.4
2	0	3.8	7.6	11.4	15.1	18.9	22.7
3	0	5.3	10.6	16.0	21.4	26.9	32.4
4	0	6.4	13.0	19.8	26.4	33.6	40.7
5	0	7.4	15.1	22.9	31.0	39.1	47.4
10	0	10.3	21.0	32.1	43.2	54.3	65.2
20	0	12.2	24.8	37.2	49.2	60.7	71.6

Effectively the results presented in Tables 2 and 3 show that the fiscal gain is indeed significant, with the fiscal gain increasing with the opportunity cost of the financial institution. However, it should be pointed out that the fiscal gain decreases when the financing interest rate i is increased.

THE EFFECT OF A SINKING FUND

With the purpose of reducing the risk of default by the borrowers, the financial institutions that offer loans is accordance with the "métodoamericano" may require that the borrowers establish a sinking fund.

Usually, the sinking fund is established with constant periodic payments, which we will denote by d, with the number of deposits being equal to the number *n*that designates the term of the loan.

Preliminarily, it should be pointed out that a loan characterizes what is said to be in Brazil, an active operation, while the deposits made by the borrowers constitute what is said to be a passive operation. Moreover, as can be seen in Banco Central do Brasil (1999), the periodic interest rate i which is charged in active operations, is usually substantially greater than the periodic interest rate $i'$of the financial institution's passive operations. For instance, in the mentioned publications, it was verified that, in monthly terms and for personal loans, on average, *i > 4.22i'* (with *i > 2.69i'*, for corporate loans).

Given that the n deposits equal to d should accumulate, at the end of the considered term, the value F of the loan, it follows that we should have:

$$\sum_{k=1}^{n} d\left(1+i'\right)^{k} = F \tag{15}$$

Therefore:

$$d = i'.F \Big/ \left\{\left(1+i'\right)^{n} - 1\right\} \tag{16}$$

From the accounting point of view, regardless of having a unique contract or *n*subcontracts, the financial institution is entitled to deduct, for income tax purposes, the parcels of interest that are credited to the sinking-fund.

Denoting by J_k' the *k*-th parcel of interest that is credited to the sinking-fund, we have that:

$$J_k' = d\left\{\left(1+i'\right)^{k-1} - 1\right\}, \text{ for } k = 1, 2, \ldots, n \tag{17}$$

In what follows, we will assume that the n deposits equal to d take place simultaneously as the payments of the loan.

A Numerical Example

As an illustration, let us revisit the loan of $ 1,000.000.00 for a term of 12 months.

If the monthly rate i is 0.5%, it follows that $d = \$81,066.43$.

Table 4 presents the evolution of the accumulated value in the sinking-fund at the end of k periods, which will be denoted by S'_k, as given by.

$$S'_k = d\left\{(1+i')^{-k} - 1\right\}/i', \text{ for } k = 1, 2, \ldots, n \tag{18}$$

Additionally, in Table 4 the evolution of each parcel of interest that is credited in the sinking-fund is presented.

Furthermore, given the results in Table 1, we also have the evolutions of the differences $J_k - J'_k$ and $\hat{J}_k - J'_k$. Obviously, we have, on the last column of Table 4, the same values presented in Table 1, for the differences $\delta_k = (J_k - J'_k) - (\hat{J}_k - J'_k)$.

Table 4. Evolution of the sinking-fund

k	S'_k	d	J'_k	$J_k - J'_k$	$\hat{J}_k - J'_k$	d_k
0	-	-	-	-	-	-
1	81,066.43	81,066.43	0.00	20,000.00	392.16	19,607.84
2	162,538.19	"	405.33	19,594.67	371.29	19,223.38
3	244,417.31	"	812.69	19,187.31	340.86	18,846.45
4	326,705.83	"	1,222.09	18,777.91	301.00	18,476.91
5	409,405.79	"	1,633.53	18,366.47	251.85	18,114.62
6	492,519.25	"	2,047.03	17,952.97	193.55	17,759.42
7	576,048.27	"	2,462.60	17,537.40	126.20	17,411.20
8	659,994.95	"	2,880.23	17,119.77	49.96	17,069.81
9	744,361.35	"	3,299.97	16,700.03	-35.08	16,735.11
10	829,149.59	"	3,721.81	16,278.19	-128.78	16,406.97
11	914,361.76	"	4,145.75	15,854.25	-231.00	16,085.25
12	1,000,000.00	"	4,571.81	15,428.19	211,165.15	-195,736.96
$\sum$	-	972,797.16	27,202.84	212,797.16	122,797.16	0.00

(Values in $)

Consolidation of the Fiscal Gain

As numerically observed on the last column of Table 4, the establishment or not of a sinking-fund does not change, from a fiscal point of view, the dominance of the policy of adopting multiple contracts, vis a vis the practice of a single contract.

That is, if we denote $V_3(\rho)$ the present value at the rate ρ of the sequence of the interest parcels, taking into consideration the joint operation, financing in accordance with the "sistemaamericano", coupled with the establishment of a sinking-fund, we have the obvious inequality: $V_1(\rho) > V_2(\rho) \Rightarrow V_1(\rho) - V_3(\rho) > V_2(\rho) - V_3(\rho)$.

As to the determination of $V_3(\rho)$, we have that:

$$V_3(\rho) = \sum_{k=1}^{n} d\left\{(1+i')^{k-1} - 1\right\}(1+\rho)^{-k} \tag{19}$$

Thus, given the values of i' and of ρ we have to distinguish the following two different possibilities:

a) $i' = \rho$

wherein

$$V_3(\rho) = d\left\{n(1+\rho)^{-1} - \left[1 - (1+\rho)^{-n}\right]/\rho\right\} \tag{20}$$

and

b) $i' \neq \rho$

wherein

$$V_3(\rho) = d\left\{\frac{1 - \left[(1+i')/(1+\rho)\right]^n}{\rho - i'} - \frac{1 - (1+\rho)^{-n}}{\rho}\right\} \tag{21}$$

As a numerical illustration, consider the case where $F = \$1,000,000.00$, $n = 60$ months, $i = 2\%$ per month, $i' = 0.08\%$ per month, and the opportunity cost for the financing institution granting the loan is ρ_a = 20% annually.

In this case, we will have that $V_1(\rho)$ = $ 781,375.72 and $V_2(\rho)$ = $ 567,999.03 which implies as shown in Table 2, that the financing institution will have a fiscal gain of 37.6%, if the effect of the constitution of sinking-fund is not taken into consideration.

On the other hand, taking into account the establishment of a sinking-fund, as $V_3(\rho)$ = $ 118,202.74 we will have that the consolidated fiscal gain will rise to 47.4%.

CONCLUSION

Similar to the cases where the loans are taken in accordance to the so called "Tabela Price" (constant payments) or in accordance with the constant amortization scheme, financing contracts that adopt the "american system of payments", where only interest is periodically paid, the financial institutions will profit if a single contract option is adopted.

The fiscal gain will be observed regardless of a sinking-fund being required for the concession of the loan or not.

REFERENCES

1. Banco Central do Brasil. *Juros e Spread Bancários no Brasil*. Brasília – DF, 1999.
2. de Faro, Clovis. *Critérios Quantitativos para Avaliação e Seleção de Projetos de Investimentos*. Rio de Janeiro: IPEA/INPES, 1971.
3. de Faro, Clovis. *On the Internal Rate of Return Criterion. Engineering Economist*, n. 19, p. 165-194, 1974.
4. de Faro, Clovis. *Matemática Financeira: uma Introdução à Análise de Risco*. São Paulo; Saraiva, 2014.
5. de Faro, Clovis Lachtermacher, *Gerson. Introdução à Matemática Financeira*. Rio/São Paulo: FGV/Saraiva, 2012.
6. de-Losso, Rodrigo; Giovannetti, Bruno Cara, & Rangel, Armênio de Souza. Sistema de Amortização por Múltiplos Contratos: a falácia do sistema francês. *Economic Analysis of Law Review*, 4, p. 160-180, 2013.
7. Halter, Bárbara. *O uso do Sistema Americano para Operações de Empréstimos eFinanciamentos.* Curso de Licenciatura em Matemática. Universidade do Estado de Santa Catarina. Centro de Ciências Tecnológicas. Joinville – SC, 2013.
8. Sandrini, Jackson C. *Sistemas de Amortizações de Empréstimos e a Capitalização de Juros*: *Análise dos Impactos Financeiros e Patrimoniais*. Universidade Federal do Paraná. Setor de Ciências Sociais Aplicadas. Mestrado em Contabilidade. Área de Concentração: Contabilidade e Finanças. Curitiba, Paraná, 2007.
9. Veras, Lilia L. *Matemática Financeira*, 2ª Ed., São Paulo: Atlas, 1991.

Citation: Clovis de Faro, "Multiple Contracts: The Case of Periodic Payments Only", American Research Journal of Business and Management, Vol 7, no. 1, 2021, pp. 1-6.