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A Note on: Optimal Stopping of Trading Strategies

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ABSTRACT

This paper is concerned with optimal time for enter or exit of a financial position such as sell or buying a specified stock. Throughout the current paper, it is assumed that the mean function of return process of asset (say stock) price is time varying. Indeed, first, considering the step function of Yao (1984), optimal trading strategies problem are studied using optimal stopping technique and dynamic programming solutions are proposed. Optimal stopping rules are presented and they are applied to two real data sets. Then, alternative ways for formulation of time varying mean functions are studied throughout the simulated examples. Finally, concluding remarks are proposed.

KEYWORDS: *Dynamic programming; Financial position; Mean function; Optimal stopping; Trading strategies*

INTRODUCTION

A trader is a person or entity, in finance, which buys and sells financial instruments such as stocks, bonds, commodities, derivatives, and mutual funds in the capacity of agent, hedger, arbitrageur, or speculator. Traders buy and sell financial instruments traded in the stock markets, derivatives markets and commodity markets, comprising the stock exchanges, derivatives exchanges, and the commodities exchanges. Several categories and designations for diverse kinds of traders are found in finance, these may include: local, floor, high-frequency, pattern day, rogue and stock traders. All types of these market participants need know when enter and when exit of a financial position. This question is answered using the optimal stopping techniques. In mathematics, the theory of optimal stopping or early stopping is concerned with the problem of choosing a time to take a particular action, in order to maximize an expected reward or minimize an expected cost. Optimal stopping problems can be found in areas of statistics, economics, and mathematical finance (related to the pricing of American options). A key example of an optimal stopping problem is the secretary problem. Optimal stopping problems can often be written in the form of a Bellman equation, and are therefore often solved using dynamic programming, see Tijms (2012) and references therein.

This paper develops some results about the optimal exit or enters to a position in a financial market. To this end, let s_t be the price of a financial asset like stock price at time t=1,…,n and $r_t=\frac{s_t-s_{t-1}}{s_{t-1}}$ be the related return process. Suppose that conditional on some processes such as μ_t, then r_t's are independent random variables with mean μ_t. Indeed, $r_t=\mu_t+\sigma z_t$, where z_t's are independent standard normal *N(0,1)* distributed random variables. Also, assume that μ_t's are independent and also mutually independent of z_t's with common distribution $N(\gamma,v^2)$. Conditional on μ_t's, it is interested to find the stopping time τ which maximizes $E(s_\tau)$. To this end, using the dynamic programming solution of above mentioned optimal stopping problem, it is seen that

$$\pi_t=\max\left(s_t,E\left(\pi_{t+1}\middle|\mathbb{F}_t\right)\right),t=1,2,\dots,n-1,$$

where $\mathbb{F}_t$ is the σ-field generated by s_i, i=1,2,…,t. Notice that $\pi_n=s_n$. Using the Markov property of s_t, it is seen that $E(\pi_{t+1}|\mathbb{F}_t)=E(\pi_{t+1}|s_t)$. By recursive solution of above equation, it is seen that $\pi_{n-1}=(1+\gamma_n^+)s_{n-1},\pi_{n-2}=(1+\gamma_{n-1}^+)s_{n-2},\dots$ at which

$$\begin{cases}\gamma_n=(1+\mu_n)-1=\mu_n\\ \gamma_{n-1}=(1+\mu_{n-1})(1+\gamma_n^+)-1\\ \gamma_{n-2}=(1+\mu_{n-2})(1+\gamma_{n-1}^+)-1\\ \gamma_{n-1}=(1+\mu_{n-3})(1+\gamma_{n-2}^+)-1\\ \vdots\end{cases}$$

Here, $x^+=\max(x,0)$. Indeed, one can see that $\pi_{n-i}=(1+\gamma^+_{n-i+1})s_{n-i}$, where $\gamma_{n-i}=(1+\mu_{n-i})(1+\gamma^+_{n-i+1})-1$ with initial value $\gamma_n=\mu_n$. The stopping time τ is the random time that $\pi_{n-\tau}=s_{n-\tau}$. Equivalently, $\gamma^+_{n-\tau+1}=0$. Let $\tau_{n-1}\geq\tau_{n-2}\geq\cdots$ be the possible values of τ and let $\lambda_{n-j}=\tau_{n-j}-\tau_{n-j+1}, j\geq 1$ and $\tau_n=n$. Clearly, λ's are independent.

The rest of paper is designed as follows. In the next section, two real data sets are analyzed by supposing a noisy step function for mean function. Then, alternative ways for formulation of time varying mean function are proposed. Section 3 concludes.

EMPIRICAL RESULTS

In this section, empirical results of above mentioned theoretical results are surveyed. This section has two parts. First, considering mean function of Yao (1984), two real data sets are analyzed. In the second sub-section, via Monte Carlo simulations, some alternative methods for modeling time varying mean functions are studied.

Real Data Sets

Here, two real data sets are analyzed. Indeed, a backward version of noisy discrete time step function of Yao (1984) is considered for modeling μ_{n-i}'s as follows, in both two real data sets:

$$\mu_{n-i} = (1 - J_{n-i})\mu_{n-i+1} + J_{n-i}z_{n-i},$$

where μ_1 is kept fixed and J_i's are independent and identically distributed Bernoulli random variables at which J_i=1 if μ_i-μ_{i-1}≠0 with probability of p and the magnitude of changes are z_i independent of J_i and have common distribution (δ,v^2).

(a) Apple Stock. The first data set contains the daily stock price of Apple Inc. for period of 7 Aug. 2018 to 28 Feb. 2020 including 393 observations. The rolling means $\hat{\mu}_i$ (across a window of length 10) are derived and its plot is given as follows. Clearly, a non-stationary pattern in mean is seen. If $|\hat{\mu}_i - \hat{\mu}_{i-1}|$>0.001, it is assumed that J_i=1 and $\hat{z}_i = \hat{\mu}_i - \hat{\mu}_{i-1}$ It is seen that p=0.644, δ=-5.023×10^{-5}, v=0.00321. Clearly, the normality of $\hat{z}_i$ is satisfied.

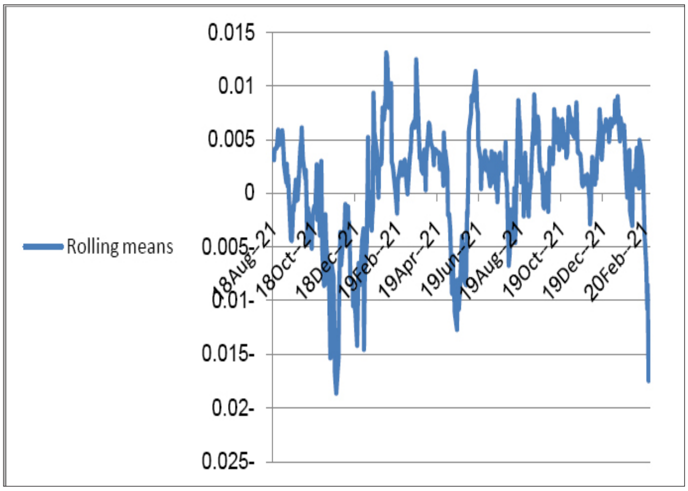

Fig. 1. Rolling means of returns

The following Table gives the mean and standard deviation of sampling first, second and third quintiles of stopping times at which λ=μ, over the sample size, i.e., τ_{n-1}/n. This Table gives useful information for traders.

Table 1. Mean and Stdev. of τ_{n-1}/n

Quartiles	Mean	Stdev.
Q1	0.597	0.282
Q2	0.718	0.239
Q3	0.857	0.164

(b) $\frac{GBP}{USD}$ Exchange rate. The second real data set contains the daily historical $\frac{GBP}{USD}$ exchange rates for study period 5 Mar. 2018 to 3 Mar. 2020 involving 521 observations. The rolling means (across a window with length 10) are plotted as follows. Clearly, a step function is fitted to the mean function of this process. It is seen that p=0.796 and z's are normally distributed with mean -0.00014 and standard deviation 0.001858.

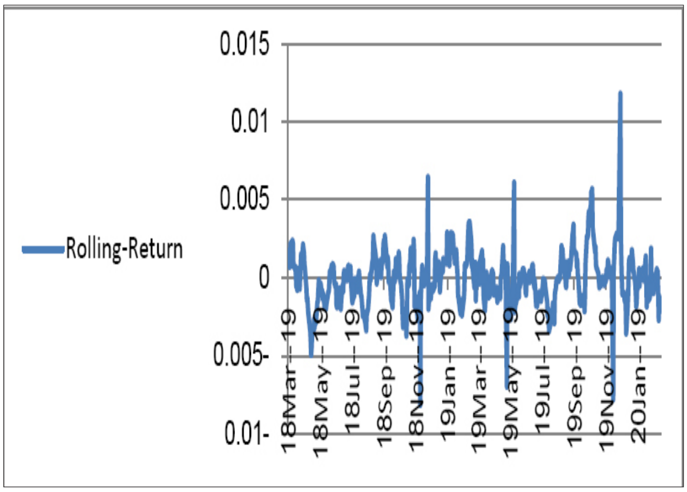

Fig.2. Rolling means

The following Table gives the mean and standard deviation of sampling first, second and third quintiles of stopping times at which λ=μ, over the sample size, i.e., τ_{n-1}/n. This Table gives useful information for traders.

Table 2: Mean and Stdev. of τ_{n-1}/n

Quartiles	Mean	Stdev.
Q1	0.397	0.238
Q2	0.629	0.197
Q3	0.801	0.165

Alternative time varying μ's

Here, by running some simulations, alternative ways for modeling time varying μ's are surveyed. These ways contain independent μ's, random walk mean function, Bayesian setting, and finally CAPM modeling of time varying μ's. All formulations propose random mean function except, the last one.

Example 1 (Independent means). Assuming μ_i,i=1,2…,n=100 are independent random variables come from normal distribution with zero mean and standard deviation 0.1, the following plot gives the time series plot of π_t-s_t,t=1,…,100. It is seen that at τ=82 93 95 96 97 98 100 the plot is zero and they are optimal time for trading.

Hereafter, a mixture distribution is fitted for τ_{n-1}/n. Notice that for small n's, it is possible that none of γ's equal to the any μ. Denote the probability of this event by π. Let, n=10, and μ's come form a standard normal distribution. Here, π=0.173 and the histogram of τ_{n-1}/n given $\tau_{n-1}<\infty$ is given as follows:

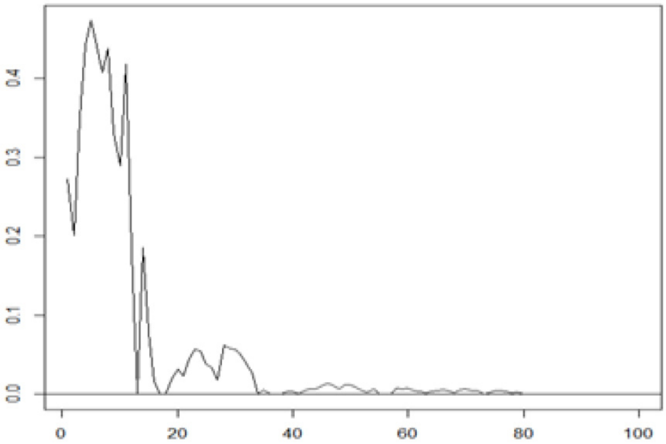

Fig. 3. Time series plot of π_t-s_t, t=1,...,100

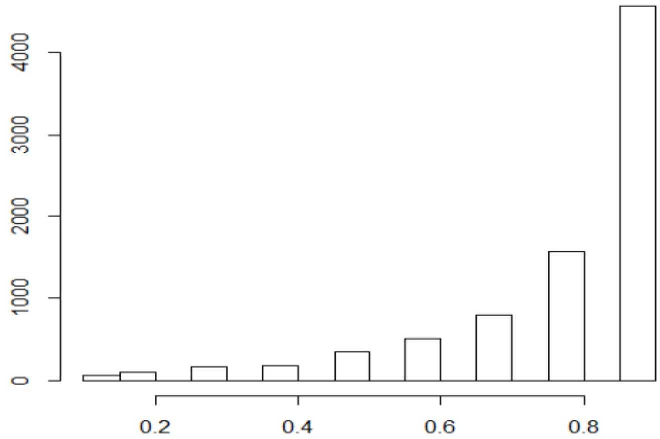

Fig. 4. Histogram of τ_{n-1}/n given $\tau_{n-1}<\infty$

Example 2 (Random walk mean process). In this simulated example, it is assumed that time varying process μ_t has a backward random walk structure, i.e.,

$$\mu_{n-t} = \mu_{n-t+1} + \zeta_{n-t}, \mu_n = \zeta_n, n = 100$$

where ζ_{n-t}'s are mutually independent and have common distribution normal distribution with zero mean and standard deviation β. Assume that β=0.1. Then, the following table gives the numbers (N) of matching μ's and γ's.

Table 3. Summary statistics of *N* (Random walk)

Mean	Stdev.	Q1	Q2	Q3	Skew	Kurt
44.14	37.3	4	38.5	79.25	0.187	1.36

Next, consider a backward stationary first order auto-regressive AR(1) process for μ_{n-t}, assuming that $\mu_{n-t}=\alpha\mu_{n-t+1}+\zeta_{n-t}$, where α=0.2. Then,

Table 4. Summary statistics of (AR(1))

Mean	Stdev.	Q1	Q2	Q3	Skew	Kurt
16.3	9.07	8	15	22	0.42	2.43

Example 3 (Bayesian setting). Here, we adopt a Bayesian time varying structure for finding μ_t. Suppose that r_t has normal distribution with mean μ_t and constant (for simplicity arguments) volatility σ (it could be supposed that volatility process obeys a heteroscedasticity process such as ARCH or GARCH series, which is omitted, here). Also, for simplicity reasons, consider a conjugate prior normal distribution for μ_t with hyper-parameters mean α and standard deviation β. Then, μ_t given r_t has normal distribution with mean $\frac{\sigma^2}{\sigma^2+\beta^2}\alpha + \frac{\beta^2}{\sigma^2+\beta^2}r_t$ and variance $\frac{\sigma^2\beta^2}{\sigma^2+\beta^2}$. Here, assume that α=0.001, β=0.012, σ=0.025. Then, the conditional distribution of μ_t given r_t is normal with mean $0.81\alpha+0.19r_t=0.19r_t+0.000813$ and standard deviation 0.01082. Generating r_t,t=1,2,...,n=100 from normal distribution with mean 0.0023 and standard deviation 0.025, the histogram of the scaled first stopping time is given as follows.

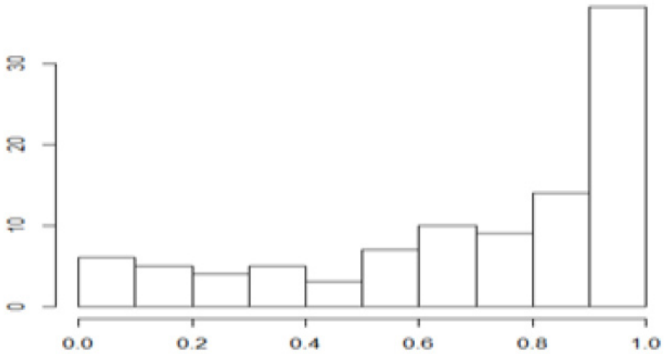

Fig. 5. Histogram of τ_1/n

Example 4 (CAPM modeling). In this example, a financial modeling of time varying μ_t is given. According to the CAPM theory the formula for calculating the expected return of an asset given its risk is as

$$\mu = r_f + \beta(\mu_{r_m} - r_f),$$

where μ, r_f, β, and μ_{r_m} - r_f are expected return of investment, risk-free rate, beta of the investment and market risk premium, respectively. Investors expect to be compensated for risk and the time value of money. The risk-free rate in the CAPM formula accounts for the time value of money. The other components of the CAPM formula account for the investor taking on additional risk. The beta of a potential investment is a measure of how much risk the investment will add to a portfolio that looks like the market. If a stock is riskier than the market, it will have a beta greater than one. If a stock has a beta of less than one, the formula assumes it will reduce the risk of a portfolio. A stock's beta is then multiplied by the market risk premium, which is the return expected from the market above the risk-free rate. The risk-free rate is then added to the product of the stock's beta and the market risk premium. Other models for estimating the price of a financial asset such as three factor models of Fama and French is not considered, here. Interested readers may refer to Glen (2005) and references therein. Following French (2016), the time varying CAPM model is given by

$$\mu_t = r_f + \beta_t(\mu_{r_m} - r_f).$$

Here, the stock price of IBM and its market S&P500 is studied during 6 Feb. 2018 to 28 Feb. 2020 including 519 observations, taken from *www.investing.com* site. The risk free rate 0.014 is assumed. Then, the sequential slope of CAPM regression is computed. The following figure shows this series.

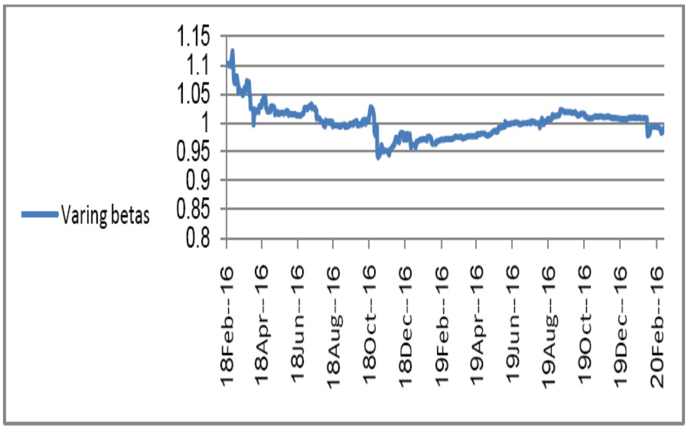

Fig. 6. Time varying betas

By Finding μ's and computing γ's, it is seen that the optimal stopping time occurs after 29 Jan 2020.

CONCLUDING REMARKS

Optimal stopping techniques, specially solved by dynamic programming in backward version, provide a useful framework for traders to decision when enter or exit of a financial position. Derived optimal stopping rules are applied to two real data sets which show the applicability of proposed technique and some other its features are presented in simulated illustrative examples.

REFERENCES

1. French, J. (2016). Estimating time-varying beta coefficients: an empirical study of US & ASEAN portfolios. *Research in Finance* 32. 19–34.
2. Glen, A. (2005). *Corporate financial management*. 3th edition. Prentice Hall. USA.
3. Tijms, H. (2012). Stochastic games and dynamic programming. *Asia Pacific Mathematics Newsletter* **3**, 6-10.
4. Yao, Y. C. (1984). Estimation of noisy discrete time step function: Bayes and Empirical Bayes approaches. *The Annals of Statistics* **12**, 1434-1447.

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